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A THEORETICAL INVESTIGATION OF THE STRUCTURE OF EASTERLY WAVES

by

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THESIS

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ABSTRACT

A simple two-level numerical model using the quasi-geostrophic forecast equations is developed. Equations are linearized and friction is introduced in the surface layer. Solutions are obtained numerically by using the initial value approach. Two wind profiles, $U = -U_o \tanh y/y_o$ and $U = U_o \operatorname{sech}^2 y/y_o$, are used and these are known to be unstable. For each wind profile the growth rate is determined as a function of the wave number. Some observed features of easterly waves are reproduced in the numerical solutions.

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TABLE OF SYMBOLS AND ABBREVIATIONS

A_e ..... kinematic eddy viscosity

α ..... inflow angle

β_o ..... derivative of coriolis parameter at $y = 0$

C_p ..... specific heat at constant pressure

f_o ..... coriolis parameter at $y = 0$

g ..... gravity

ω ..... dp/dt

Ψ ..... gz/f_o

R ..... gas constant

ρ ..... density

$\overline{T}$ ..... average temperature from some standard atmosphere

σ ..... $(R^2\overline{T}/p^2g)\ (\partial\overline{T}/\partial z + g/C_p)$

ζ ..... $\nabla^2\Psi$

z ..... height

ACKNOWLEDGMENTS

The author wishes to express his deep gratitude to Dr. Roger Terry Williams for without his patience, guidance, encouragement and counsel this thesis would have been infinitely more difficult.

A sincere thank you is also offered to Captain Thomas Kent Schminke, USAF, who cheerfully allowed his basic model to be modified for the experiments conducted.

1. INTRODUCTION

Synoptic scale disturbances in the tropical atmosphere have long been recognized (Palmer, 1951; Riehl, 1954). In general these disturbances possess a "cold-core" structure with the colder air and most active weather located east of the wave (Yanai and Nitta, 1968b). Vertical motions associated with these waves are generally upward east of the wave axis and downward to the west (Yanai and Nitta, 1967). The source of energy for these waves is not fully known, however, Charney (1963) argues that when condensation is absent the large scale motions tend to be quasi-horizontal and quasi-nondivergent. If this argument is correct then the barotropic instability that arises as the result of horizontal shear in the Intertropical Convergence Zone could be the source of energy for the disturbances (Schminke, 1968).

Kuo (1949) studied the stability characteristics of a barotropic zonal current, and showed that the barotropic zonal current is stable if the gradient of absolute vorticity has the same sign in the zonal current. Charney (1964) discussed the case where horizontal shear instability is produced in the equatorial convergence zone when the zone is located away from the equator and the converging air masses carry their angular momentum with them.

The purpose of this paper is to devise and test a numerical model that is structured realistically, that is, the perturbation has a configuration and behavior similar to observed easterly waves. The main feature desired is to have the associated weather, and therefore the vertical motion, upwind of the trough axis.

2. BASIS OF THE MODEL

The assumption that the source of energy for the wave mechanism is barotropic instability leads to the selection of a basic current that is independent of height. Baroclinicity has been omitted but the atmosphere remains stratified. Charney (1963) used a scale analysis to show that the barotropic vorticity equation governs tropical motions in the absence of condensation. In another paper Charney (1969) discusses the role of the phase velocity in energy propagation in an easterly regime. Following Schminke (1968) the assumption is made that the easterly waves can be roughly described by a two-level quasi-geostrophic model. It is recognized that the quasi-geostrophic approximation is not very accurate close to the equator, however, the zone of dynamic instability is assumed to be associated with the Intertropical Convergence Zone which is some distance from the equator so that the approximation is relatively accurate.

The type of wind profile chosen will play a large role in determining the stability of the disturbance. Jacobs and Wiin-Nielsen (1966) investigated barotropic instability and found that there are several unstable modes. Since a basic zonal current with no change in the vertical is used in this model, the wave with no vertical motion would predominate (Jacobs and Wiin-Nielsen, 1966). Easterly waves are observed to have vertical motion, so a method must be found to introduce it. This is accomplished by introducing friction in the surface boundary layer. Thus vertical motion, which depends upon the stratification of the atmosphere, is now forced into the model.

3. THE FORECAST EQUATIONS

A simple two-level model is constructed by dividing the entire atmosphere into four layers of constant pressure differential, $\Delta p/2$ (fig. 1), numbered 0 to 4 from top to bottom. Vertical motion is assumed to be zero at the

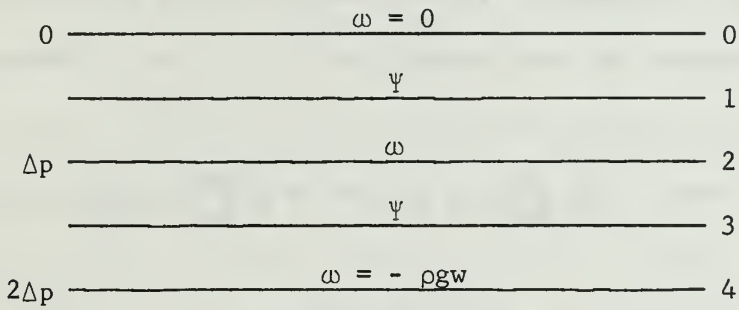

Fig. 1. Two-level model used for prediction.

top of the atmosphere, while the vertical motion term at the earth's surface is

$$\omega \cong -\rho g w_4 . \tag{3.1}$$

Charney and Eliassen (1949) used the Ekman theory to derive an expression for w_4 which is

$$w_4 = \frac{1}{2}\left(\frac{2A_e}{f_o}\right)^{1/2} \sin 2\alpha\, \zeta_4 , \tag{3.2}$$

where A_e is the kinematic eddy viscosity and α, the inflow angle. The surface geostrophic vorticity can be approximated by

$$\zeta_4 \cong \zeta_3 = \nabla^2 \Psi_3 . \tag{3.3}$$

Following the development of Thompson (1961), but using different notation, begin with the quasi-geostrophic vorticity equation

$$\frac{\partial}{\partial t}\nabla^2\Psi + \mathbf{k} \times \nabla\Psi\cdot\nabla(\nabla^2\Psi) + \beta_o\frac{\partial\Psi}{\partial x} - f_o\frac{\partial\omega}{\partial p} = 0. \tag{3.4}$$

Notice that when $\omega = 0$, the above equation becomes the barotropic vorticity equation, which is sufficient to describe the main instability.

Apply this equation at levels 1 and 3 giving

$$\frac{\partial}{\partial t}\nabla^2\Psi_1 + |k \times \nabla\Psi_1 \cdot \nabla(\nabla^2\Psi_1) + \beta_o \frac{\partial\Psi_1}{\partial x} - f_o \frac{\omega_2}{\Delta p} = 0, \tag{3.5}$$

$$\frac{\partial}{\partial t}\nabla^2\Psi_3 + |k \times \nabla\Psi_3 \cdot \nabla(\nabla^2\Psi_3) + \beta_o \frac{\partial\Psi_3}{\partial x} + f_o \frac{(\omega_4 - \omega_2)}{\Delta p} = 0. \tag{3.6}$$

Smaller layers could be used if it is desired to stop at some intermediate point, say the tropopause, rather than including the entire atmosphere.

Next consider the quasi-geostrophic first law of thermodynamics in the form

$$\frac{\partial}{\partial t}\frac{\partial\Psi}{\partial p} + |k \times \nabla\,\Psi \cdot \nabla\left(\frac{\partial\Psi}{\partial p}\right) + \frac{\sigma}{f_o}\,\omega = 0, \tag{3.7}$$

where

$$\sigma = \frac{R^2\bar{T}}{p^2 g}\left[\frac{\partial\bar{T}}{\partial z} + \frac{g}{C_p}\right]. \tag{3.7a}$$

When this equation is applied at level 2 the result is

$$\frac{\partial}{\partial t}(\Psi_1 - \Psi_3) + |k \times \nabla\frac{(\Psi_1+\Psi_3)}{2} \cdot \nabla(\Psi_1-\Psi_3) - \frac{\Delta p\sigma\omega_2}{f_o} = 0. \tag{3.8}$$

Define the following quantities

$$\Psi_m = \frac{\Psi_1 + \Psi_3}{2}, \tag{3.9}$$

$$\Psi_T = \frac{\Psi_1 - \Psi_3}{2}, \tag{3.10}$$

which implies that

$$\Psi_1 = \Psi_m + \Psi_T \tag{3.11}$$

$$\Psi_3 = \Psi_m - \Psi_T. \tag{3.12}$$

Here Ψ_T is proportional to the layer thickness, therefore is a measure of its mean temperature. Using these definitions add (3.5) and (3.6) and divide the result by 2, obtaining

$$\frac{\partial}{\partial t}\nabla^2\Psi_m + |k \times \nabla\Psi_m \cdot \nabla(\nabla^2\Psi_m) + |k \times \nabla\Psi \cdot \nabla(\nabla^2\Psi_T) + \beta_o \frac{\partial\Psi_m}{\partial x} - f_o \frac{\omega_4}{2\Delta p} = 0. \tag{3.13}$$

For the second forecast equation subtract (3.6) from (3.5) and eliminate ω_2 using (3.8)

$$\frac{\partial}{\partial t}(\nabla^2 - \mu^2)\,\Psi_T + \mathbf{k} \times \nabla\Psi_m \cdot \nabla(\nabla^2 - \mu^2)\,\Psi_T + \mathbf{k} \times \nabla\Psi_T \cdot \nabla(\nabla^2\Psi_m) + \beta_o \frac{\partial \Psi_T}{\partial x} + f_o \frac{\omega_4}{2\Delta p} = 0, \tag{3.14}$$

where

$$\mu^2 = \frac{2 f_o^2}{\Delta p^2 \sigma}. \tag{3.14a}$$

These are the prediction equations for the model. They can be linearized by separating the flow into an east-west current that varies only in y, and a small departure from this flow. In this event it is possible to treat various waves in x independently, hence the fields may be defined as follows

$$\Psi_m = E(y) + A(y,t)\cos kx + B(y,t)\sin kx, \tag{3.15}$$

$$\Psi_T = C(y,t)\cos kx + D(y,t)\sin kx, \tag{3.16}$$

where k is the x wave number.

Substitute expressions for Ψ_m and Ψ_T into (3.13) and (3.14), separate the various sine and cosine terms, neglecting all products of the quantities A through D, and equating coefficients of the cosine kx terms gives

$$\frac{\partial}{\partial t}\left(\frac{\partial^2 A}{\partial y} - Ak^2\right) = k\left[\frac{\partial E}{\partial y}\frac{\partial^2 B}{\partial y^2} - \frac{\partial E}{\partial y} Bk^2 - \frac{\partial^3 E}{\partial y^3} B\right] - \beta_o Bk - K\left(\frac{\partial^2}{\partial y^2} - k^2\right)(A - C). \tag{3.17}$$

For the sine terms the result is

$$\frac{\partial}{\partial t}\left(\frac{\partial^2 B}{\partial y} - Bk^2\right) = k\left[\frac{\partial E}{\partial y} Ak^2 - \frac{\partial E}{\partial y}\frac{\partial^2 A}{\partial y^2} + \frac{\partial^3 E}{\partial y^3} A\right] + \beta_o Ak - K\left(\frac{\partial^2}{\partial y^2} - k^2\right)(B - D). \tag{3.18}$$

Repeating the procedure for (3.16) the final equations are

$$\frac{\partial}{\partial t}\left(\frac{\partial^2 C}{\partial y^2} - Ck^2 - C\mu^2\right) = k\left[\frac{\partial E}{\partial y}\frac{\partial^2 D}{\partial y^2} - \frac{\partial E}{\partial y} D\,(k^2 + \mu^2) - \frac{\partial^3 E}{\partial y^3} D\right]$$

$$- \beta_o\, Dk + K\left(\frac{\partial^2}{\partial y^2} - k^2\right)(A - C), \tag{3.19}$$

and

$$\frac{\partial}{\partial t}\left(\frac{\partial^2 D}{\partial y^2} - Dk^2 - D\mu^2\right) = k\left[\frac{\partial E}{\partial y} C\,(k^2 + \mu^2) - \frac{\partial E}{\partial y}\frac{\partial^2 C}{\partial y^2} + \right.$$

$$\left.\frac{\partial^3 E}{\partial y^3} C\right] + \beta_o\, Ck + K\left(\frac{\partial^2}{\partial y^2} - k^2\right)(B - D), \tag{3.20}$$

where

$$K = \frac{f_o g}{2R\overline{T}}\left(\frac{A_m}{f_o}\right)^{1/2}. \tag{3.21}$$

It is now necessary to derive the computational equation for vertical motion starting with

$$\omega_2 = \frac{2}{\Delta p\sigma}\left[\frac{\partial\Psi_T}{\partial t} + u_m\frac{\partial\Psi_T}{\partial x} + v_m\frac{\partial\Psi_T}{\partial y}\right], \tag{3.22}$$

where

$$u_m = -\frac{1}{f_o}\frac{\partial\Psi_m}{\partial y}, \tag{3.23}$$

$$v_m = \frac{1}{f_o}\frac{\partial\Psi_m}{\partial x}. \tag{3.24}$$

Substituting equations (3.23), (3.24), (3.15) and (3.16) into (3.22), collecting terms, and again neglecting all products of the quantities A through D results in

$$\omega_2 = \frac{2}{\Delta p\sigma}\left[\cos\ kx\left(\frac{\partial C}{\partial t} - \frac{k}{f_o}\frac{\partial E}{\partial y} D\right) + \right.$$

$$\left.\sin\ kx\left(\frac{\partial D}{\partial t} + \frac{k}{f_o}\frac{\partial E}{\partial y} C\right)\right]. \tag{3.25}$$

This is the equation used in the model for the computation of vertical motion. The prediction equations and the vertical motion equation are now in the form to be used in the model.

4. BOUNDARY CONDITIONS AND FINITE DIFFERENCING SCHEME

The finite differencing scheme used is illustrated below with a sample variable N;

$$\frac{\partial N}{\partial y} = \frac{1}{2H}\left(N_{i+1} - N_{i-1}\right) \tag{4.1}$$

$$\frac{\partial^2 N}{\partial y^2} = \frac{1}{H^2}\left(N_{i+1} - 2N_i + N_{i-1}\right) \tag{4.2}$$

$$\frac{\partial^3 N}{\partial y^3} = \frac{1}{2H^3}\left[\left(N_{i+2} - 2N_{i+1} + N_i\right) - \left(N_i - 2N_{i-1} + N_{i-2}\right)\right] \tag{4.3}$$

where i is the grid index and H is the distance between grid points.

Centered time differences are used for all quantities except those involving friction. The frictional terms are computed at time $(t - \Delta t)$. The first step in all cases is a forward time step. The second order equations for time tendencies are solved by the exact method of Richtmyer (1957, p. 101).

Yanai and Nitta (1968a) studied the problem of finite difference approximations in solving dynamic instability problems of non-divergent barotropic currents. They showed that for both symmetric and anti-symmetric zonal currents the exact boundary conditions can be replaced by rigid boundary conditions at a distance equal to the half width of the shearing wind belt. At any boundary distance larger than this value the numerical and theoretical values tended to have a constant difference. Therefore rigid boundaries are placed at $y = -w/2$ and $y = w/2$, where w is the total width over which computations are made. With the exception of E (the basic current) all variables have the boundary condition $A = B = C = D = 0$ at $y = -w/2$ and at $y = w/2$. When second derivatives are required on the boundary they are set equal to zero.

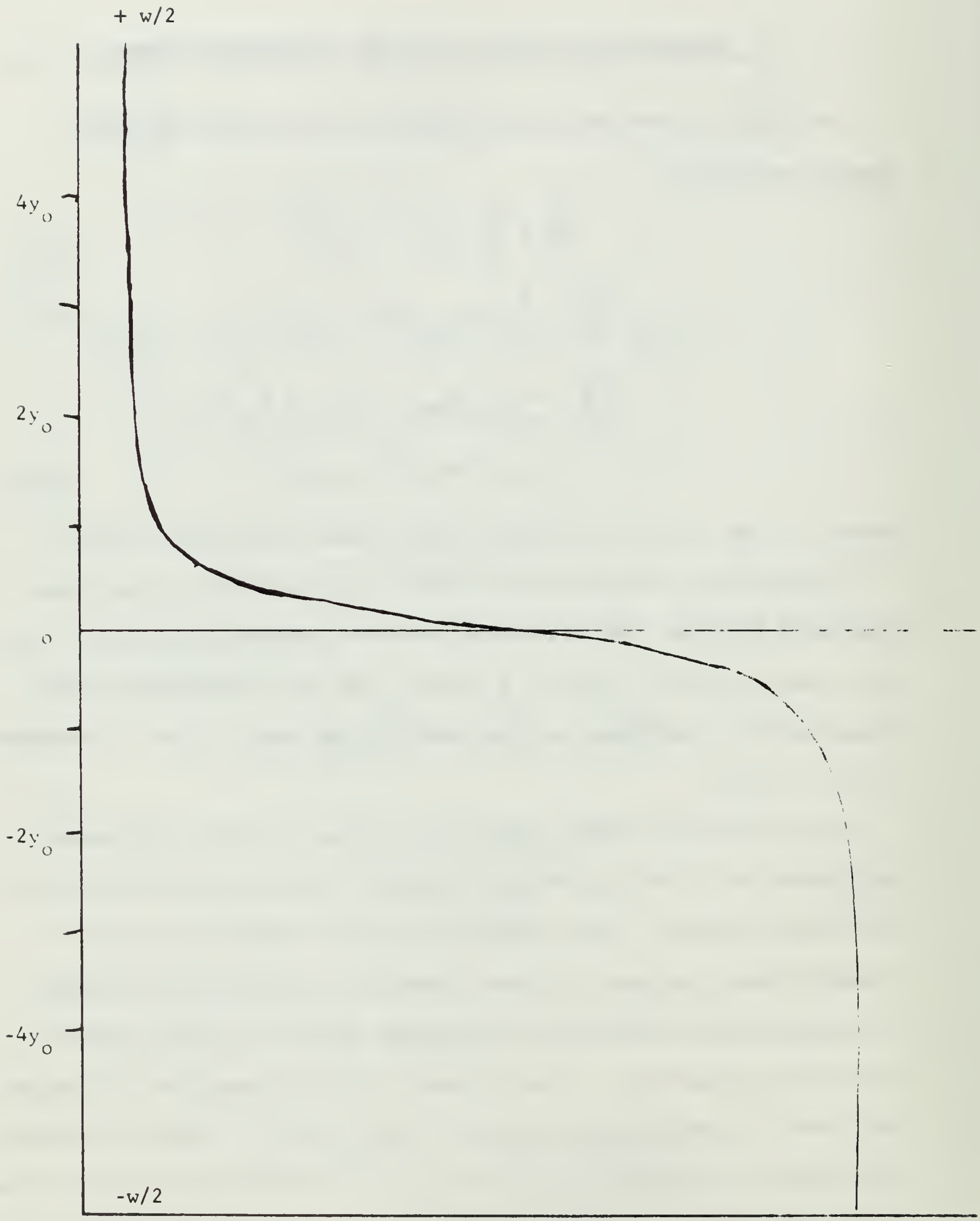

Fig. 2. Wind profile $U = -U_o \tanh y/y_o$.

5. WIND PROFILES AND INITIAL CONDITIONS

Schminke (1968) investigated and discussed the structure of easterly waves from a similar model using one wind profile ($U = -U_0 \tanh(y/y_o)$) and one non-dimensional wave number ($1/2y_o$). It is proposed to expand this to include another profile ($U = U_o \mathrm{sech}^2 y/y_o$). Additionally the behavior of disturbances at various non-dimensional wave numbers under both profiles will be considered.

Charney (1963) shows by scale analysis that in the absence of condensation, flow in the tropics is governed by the barotropic vorticity equation. The assumption is made that the source of energy for easterly waves must be barotropic instability. Since potential energy is assumed not to be an important energy source the basic temperature field may be set equal to a constant and is invariant. However, small temperature fluctuations are observed with easterly waves, so a two level model is needed (Schminke, 1968).

If the waves are to grow, the zonal wind profile must be barotropically unstable. Following the suggestion of Charney (1963) that the shear layer near the Intertropical Convergence Zone may be formed by transport of angular momentum, the unstable profile chosen to approximate this is

$$U = -U_0 \tanh y/y_o, \tag{5.1}$$

illustrated in figure 2. This seems most appropriate during monsoonal conditions. The second profile chosen,

$$U = U_o \mathrm{sech}^2 y/y_o , \tag{5.2}$$

is probably more applicable to mean flow conditions, and is illustrated in figure 3.

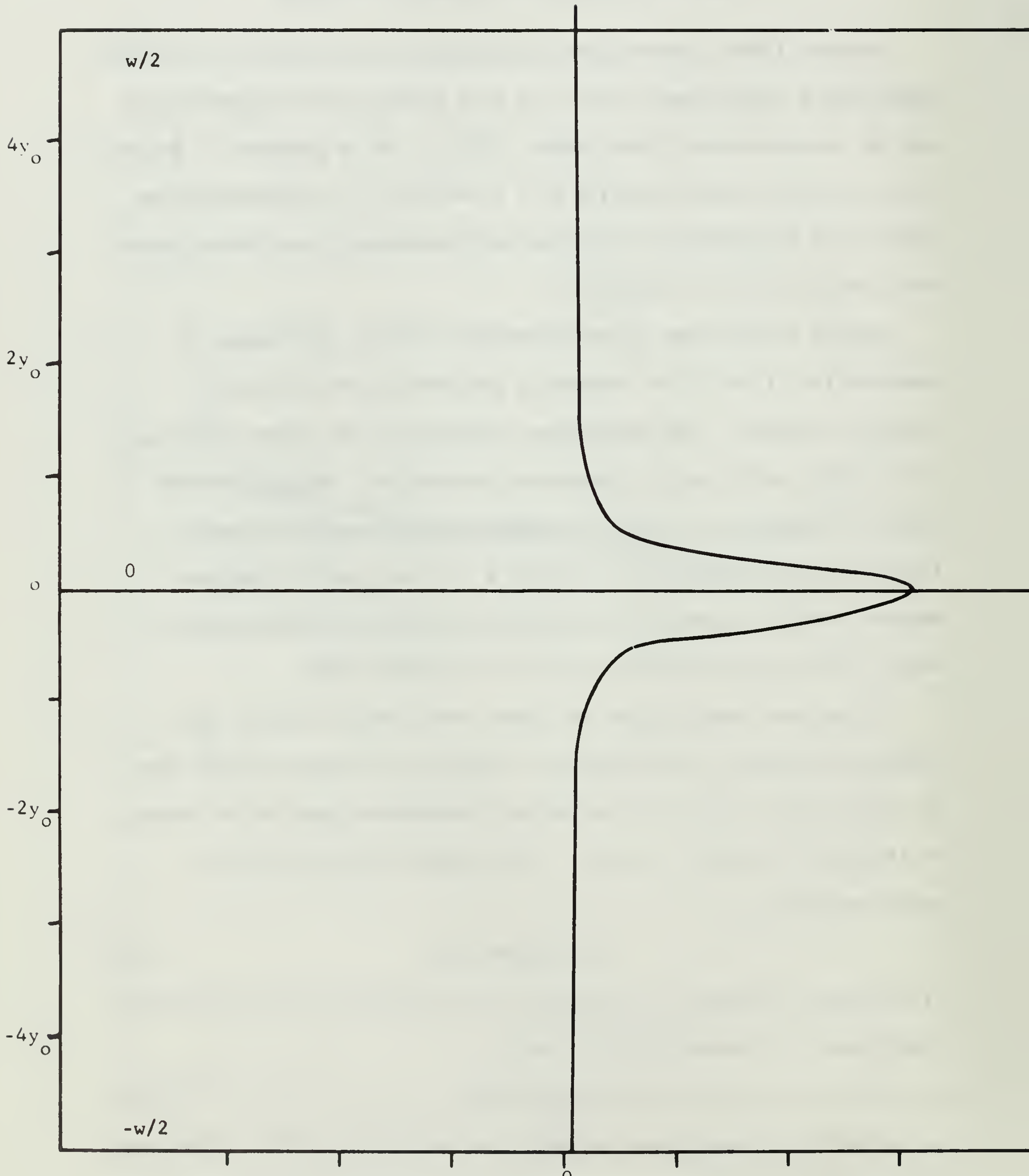

Fig. 3. Wind profile $U = U_o \mathrm{sech}^2 y/y_o$

Garcia (1956) found that the profile given in (5.1) is barotropically unstable when $0 \leq k \leq y_o^{-1}$. Betchov and Criminale (1967) have analytically, and by use of the eigenvalue approach, investigated this profile throughout the range mentioned above and found the maximum instability at a non-dimensional wave number of 0.45 ($ky_o = 0.45$). With these results in mind this paper is confined to the range 0.25 to 0.60 for the non-dimensional wave number when dealing with the hyperbolic tangent profile (5.1). Betchov and Criminale (1967) found a rather flat zone of maximum instability for the hyperbolic secant squared profile (5.2) in the range 0.80 to 1.05 for the non-dimensional wave number. Some computations were done in the lower range (0.25 to 0.60), but the majority of the experiments with this profile were confined to the range 0.70 to 1.15. Results of experiments with both profiles will be discussed in a later section.

With the wind profile independent of height and with no friction Ψ_m and Ψ_T should behave independently. Jacobs and Wiin-Nielsen (1966) showed that the growth rate for the Ψ_m field is greater than that for the Ψ_T field. The thermal field Ψ_T is zero in the initial state but it changes as a result of the Ekman friction term. Ψ_m is introduced into the model as a random disturbance in y. Introduction of surface friction leads to the coupling of the two fields. Thus at time t = 0

$$A = \text{random disturbance } (y), \tag{5.3}$$

$$B = \text{random disturbance } (y), \tag{5.4}$$

$$C = 0 \tag{5.5}$$

$$D = 0 \tag{5.6}$$

$$E = U_o y_o \ln(\cosh(y/y_o)), \tag{5.7}$$

or

$$E = U_o y_o \tanh y/y_o . \tag{5.8}$$

6. COMPUTATIONAL PROCEDURE

A list of the constants used in the computations is given below;

y_o = 200 kilometers,

w = 2000 or 4000 kilometers,

U_o = 10 meters per second,

$f_o = 5 \times 10^{-5}$ per second,

σ = 0.8 meters per second squared per centibar squared,

$\beta = 0.0$ or 2.29×10^{-11} per meter per second,

Δp = 50 centibars,

A_e = 10 meters per second squared,

α = 22.5 degrees,

$\mu^2 = 2.49 \times 10^{-12}$ per meter squared,

H = 25 or 50 kilometers,

Δt = 0.5 or 1.0 hours.

When w = 2000 km and H = 50 km a forecast period of 14 days was used, in all other cases the forecast period was 28 days. A sample program as developed by Schminke (1968) and modified to fit the IBM 360 computer is included as Appendix 1 (note: this program is for w = 2000 km and H = 25 km).

The random disturbance introduced into the Ψ_m field, (5.3) and (5.4), was generated by a random number generation program, then read directly into the computer. The same disturbance field was used for all experiments with both variations of β, H, Δt, and all variations of the non-dimensional wave number. After a few days the disturbance adjusted to an exponential growth. For convenience, growth rates were computed over the last three days of the forecast period. Once the field has adjusted it does not matter when computed, rates are always

the same. This was done by assuming the amplitudes could be written as

$$x_1 = S\exp\,(U_o n/y_o)\; t_1, \tag{6.1}$$

$$x_2 = S\exp\,(U_o n/y_o)\; t_2, \tag{6.2}$$

where n is the growth rate and S is a constant. By forming the ratio x_2/x_1, the growth rate can be shown to be

$$n = \frac{y_o \ln\,(x_2/x_1)}{U_o\,(t_2 - t_1)}\ . \tag{6.3}$$

7. RESULTS

A series of experiments was conducted under varied conditions. In general the total distance over the grid was 2000 km, but one series was done over a total distance of 4000 km. Grid mesh was either 50 km or 25 km with the smaller distance producing better results. When the grid mesh was 25 km a time step of 0.5 hours was used, however the majority of experiments were run at t = 1.0 hours. All cases were run with $\beta = 0$, then with $\beta = 2.29 \times 10^{-11}$ meters per second. Many combinations of friction, wave number and wind profile were run, and selected ones are illustrated in figures 4 through 19.

Growth rates for the disturbances with both profiles and for various non-dimensional wave numbers are shown in figures 4 through 11. Betchov and Criminale (1967) found a maximum growth rate of approximately 0.19 for a non-dimensional wave number of 0.45 using the hyperbolic tangent profile. With rigid boundaries, no friction and a fairly large grid size (H = 25 km) through the maximum shear zone (width approximately 400 km), the result (fig. 7) was a growth rate of 0.165 at a wave number of 0.45. According to Yanai and Nitta (1968a) the best results should be obtained with a minimum of 20 grid steps across the zone of maximum shear. Therefore the results for the hyperbolic tangent profile are considered good, and the trend upward toward the Betchov and Criminale (1967) results is evident as the grid size is decreased (compare figures 5 and 7). The inclusion of the β parameter had no significant effect on the growth rates giving only a slight decrease of 0.01 or less.

The hyperbolic secant squared profile results were equally encouraging. Just as Betchov and Criminale (1967) lead one to expect, a relatively broad zone of maximum growth is found reaching from 0.85 to 0.95 non-dimensional wave numbers. It is difficult to ascertain just what maximum rate Betchov and Criminale (1967) found, however, results of these experiments show rates that are significantly smaller than the rates for the hyperbolic tangent profile. The inevitable conclusion is that the hyperbolic secant squared profile is not quite as unstable as the hyperbolic tangent profile. As before, decreasing the grid size in the maximum shear area led to larger growth rates. Figures 9 and 10 illustrate this very well.

In figures 12 through 15 the amplitude and phase relationships of Ψ_m, Ψ_T and ω are depicted for the wave number of maximum growth. Amplitudes of Ψ_m, Ψ_T and ω show maxima in the region of greatest horizontal shear. The influence of the frictional term is clearly seen in the relative amplitudes of Ψ_1 and Ψ_3. The three fields show close phase relationships and all phases change rapidly through the shear zone. It appears to be a safe assumption that the temperature and pressure fields are in close relation for both levels. The omega amplitude (fig. 14) shows a distinct minimum in the center of the field but sharp maximum peaks on each side in the maximum shear zone. Phase relations between Ψ_3 and ω might almost be termed ideal with the sinking motion north of the shear zone about 70 degrees upwind of the amplitude ridge, placing the convergence in the area where it is normally observed in easterly waves.

Addition of the β parameter made small insignificant changes in the amplitude; phase relationships maintained the same basic pattern as

without β. Variation in the non-dimensional wave number over the range 0.25 to 0.60 left the pattern basically unchanged while producing the growth rate changes discussed above.

Figures 16 through 19 illustrate the behavior of the hyperbolic secant squared profile. Note that Ψ_1 far exceeds the other components in amplitude (fig. 16) and that the maximum is symmetrical around the center of the maximum shear zone. Ψ_3 is very small indicating that friction is probably more effective with the hyperbolic secant squared profile. The lower field, Ψ_3, tends to lead the upper field in westward movement. This appears reasonable and is observed in tropical disturbances of this nature (Riehl, 1954, pp. 215). This gives the entire system a tilt toward the east with increasing height; this tilt is not noticeable with the hyperbolic tangent profile.

The omega amplitude is very sharp peaked and symmetric about the center. Velocities, from about $\pm\, y_o$ to the boundaries, are very small. In this area the convergence appears to be about 100 degrees out of phase with the trough. Near $y_o = 0$, along the Intertropical Convergence Zone, the convergence actually leads the trough.

Use of a finite value for the β parameter made no significant difference in the results with this profile.

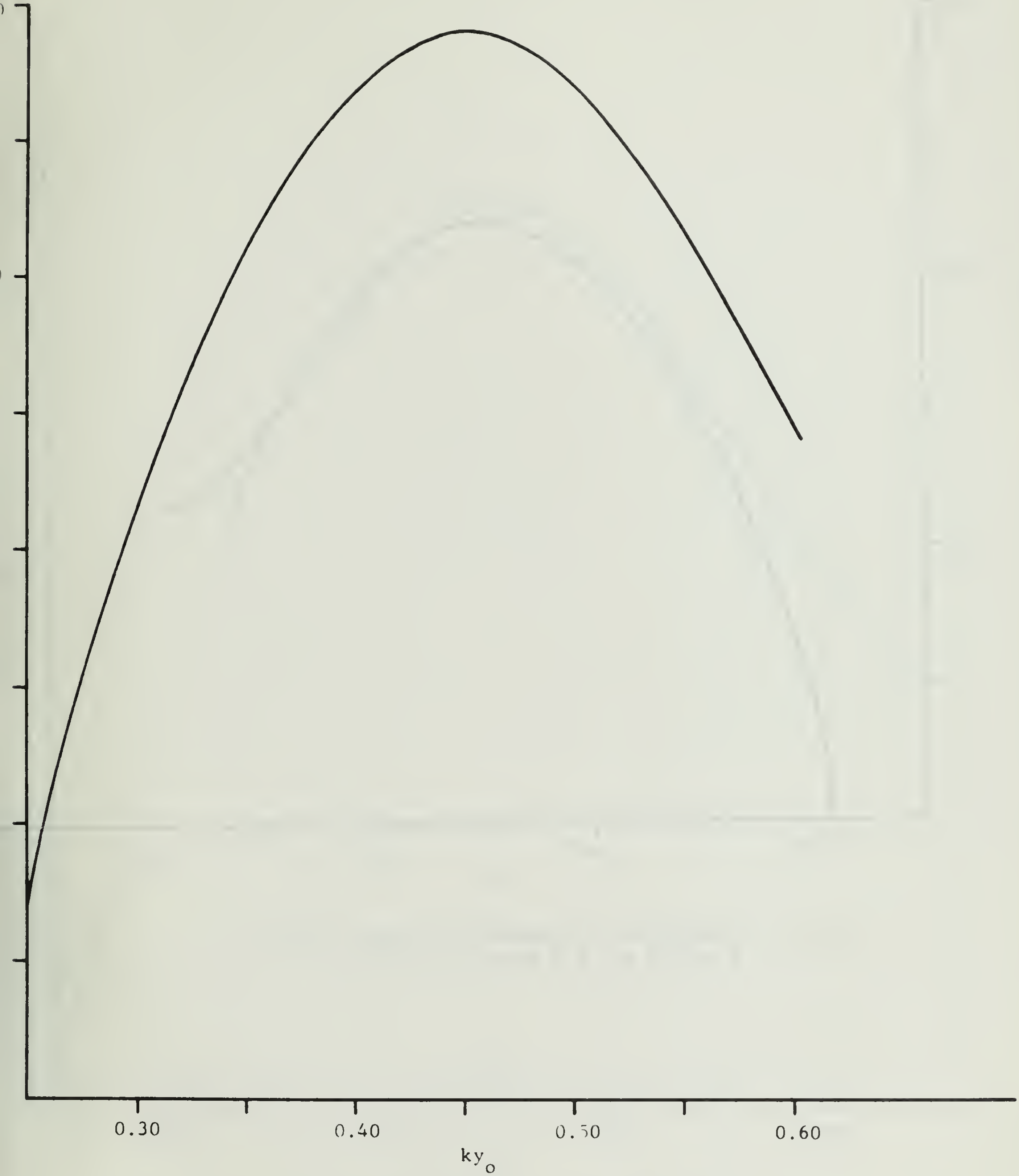

Fig. 4. Growth rate for hyperbolic tangent profile, w = 2000 km, H = 50 km, with friction.

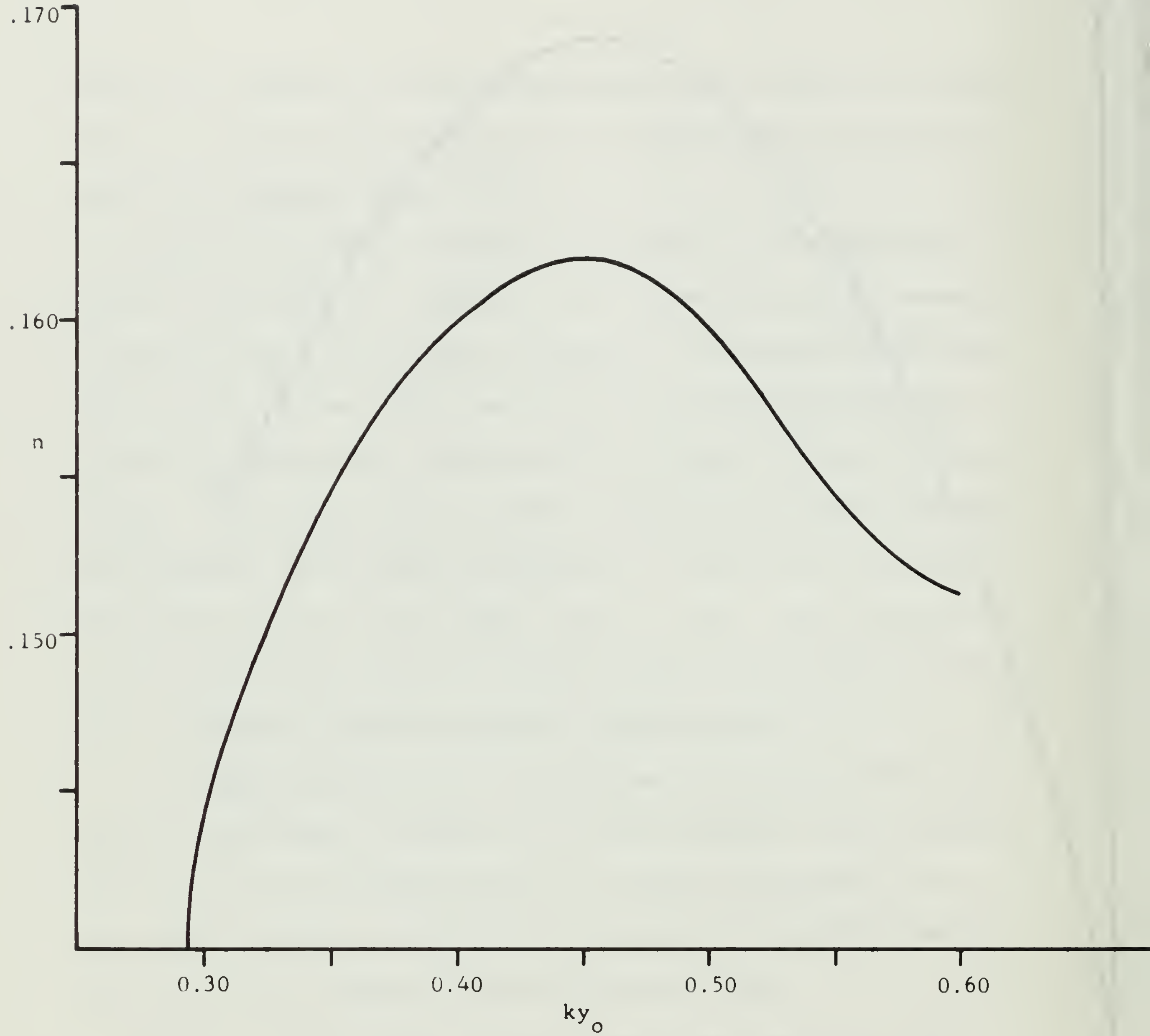

Fig. 5. Growth rate for hyperbolic tangent profile, w = 2000 km, H = 50 km, no friction.

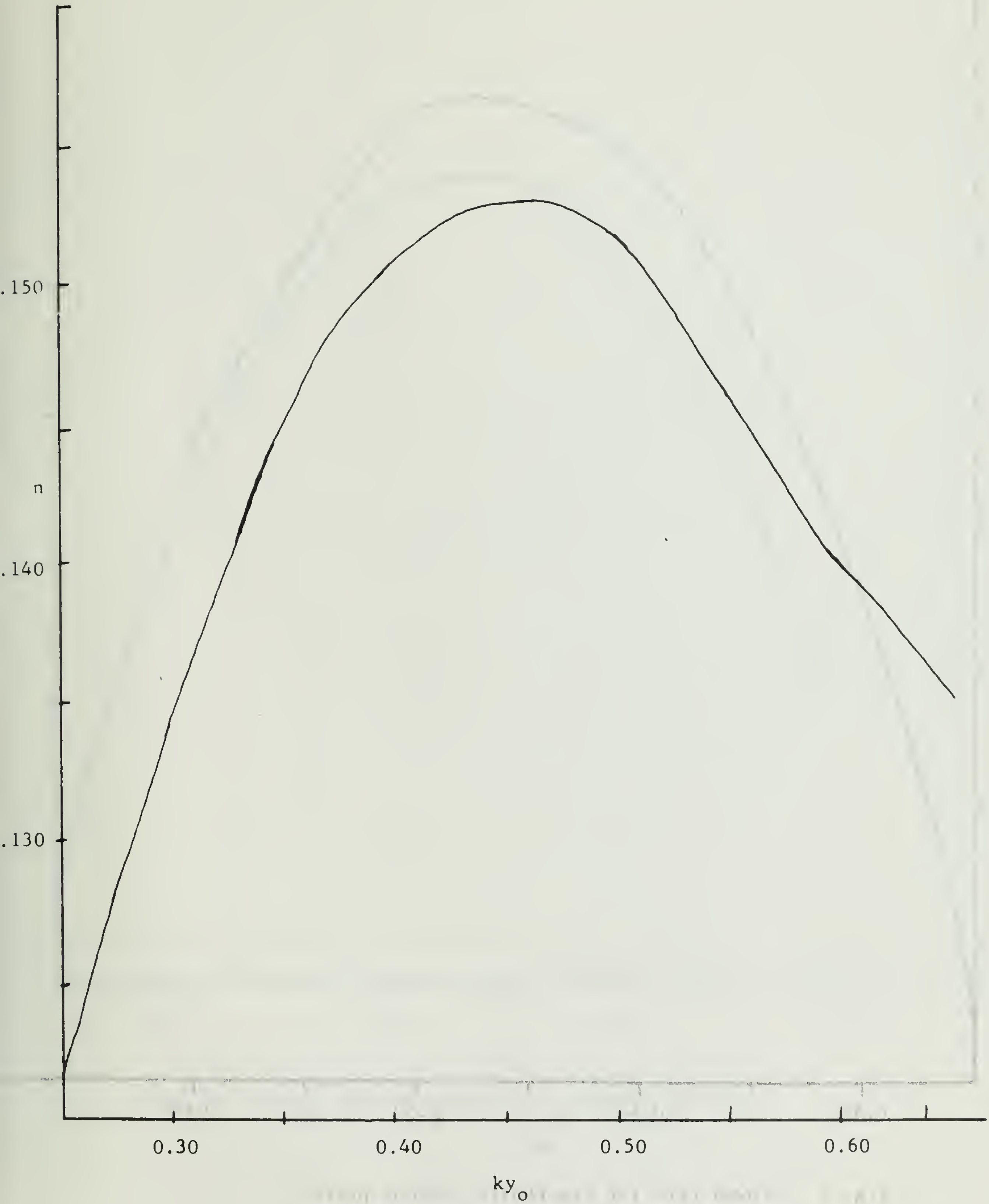

Fig. 6. Growth rate for hyperbolic tangent profile, w = 2000 km, H = 25 km, with friction

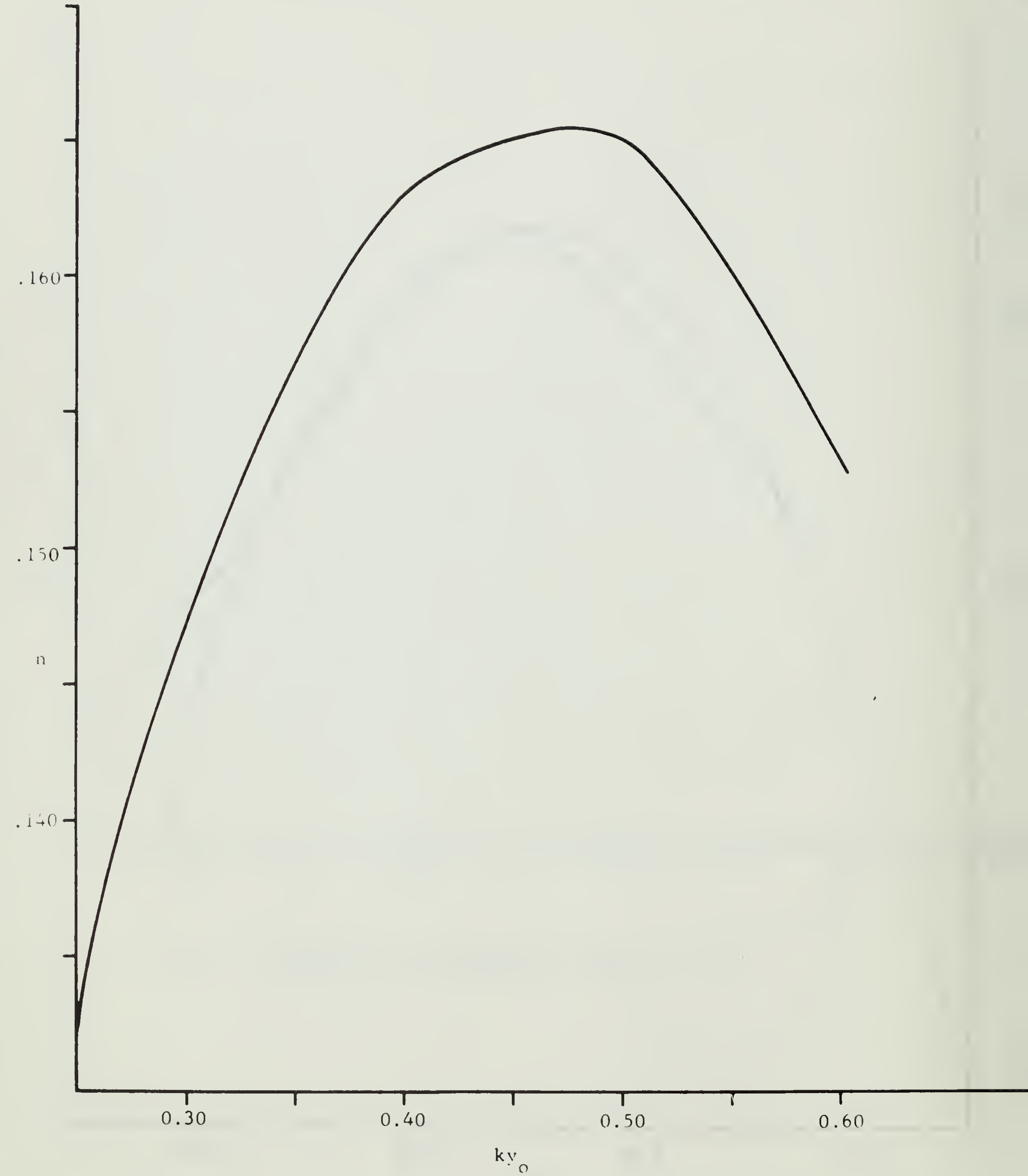

Fig. 7. Growth rate for hyperbolic tangent profile, w = 2000 km, H = 25 km, with no friction.

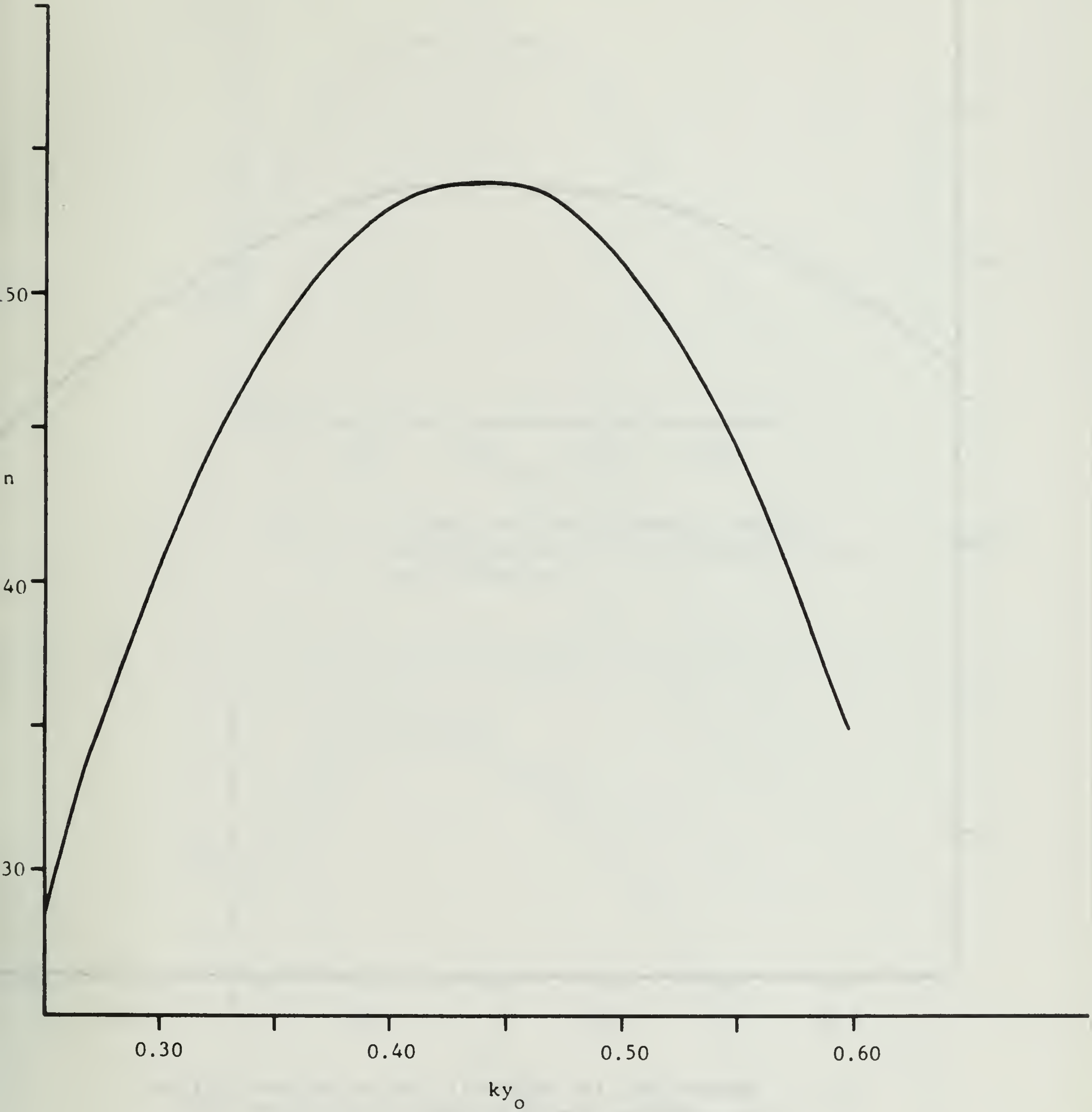

Fig. 8. Growth rate for hyperbolic tangent profile, w = 4000 km, H = 50 km, with friction.

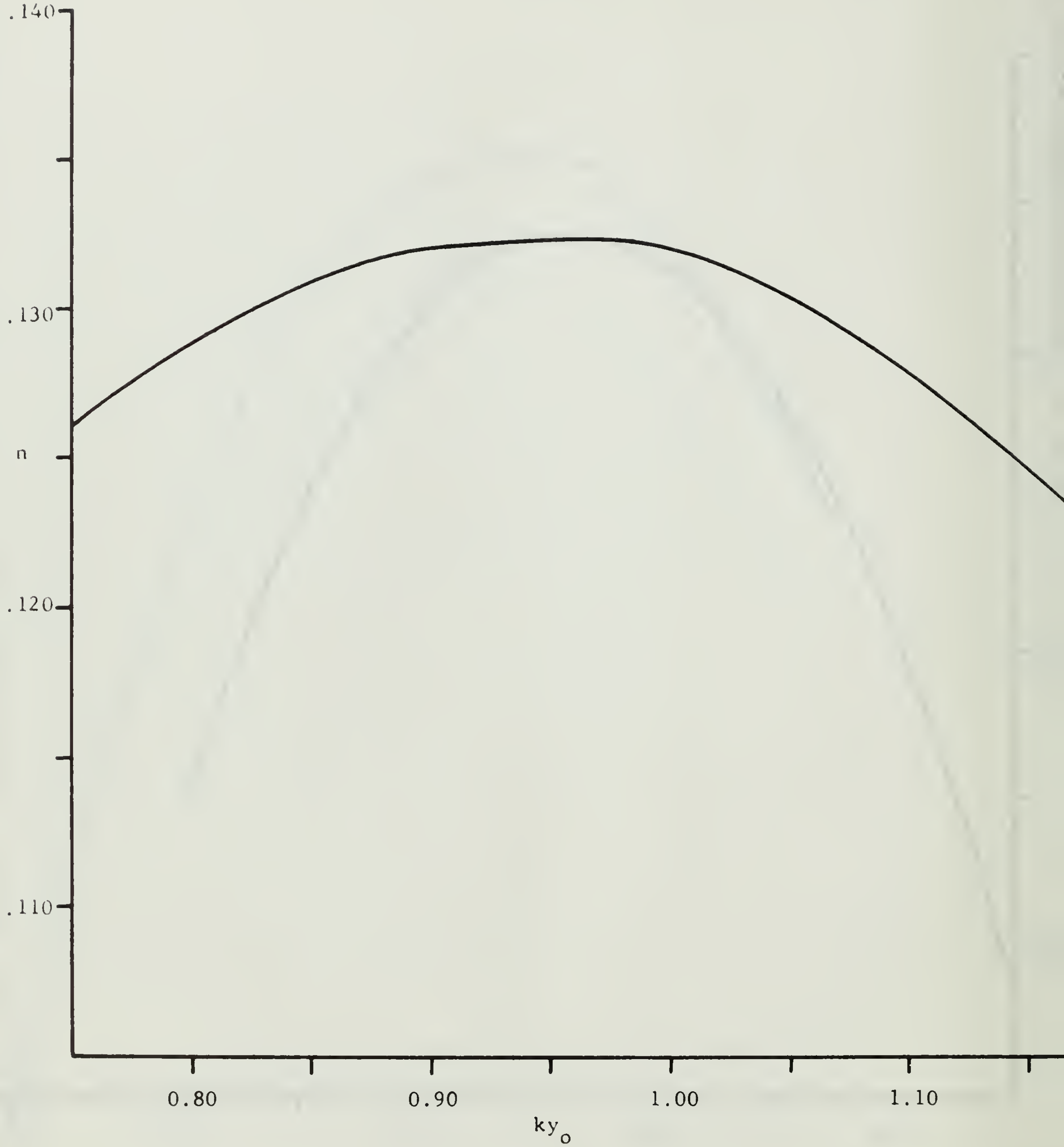

Fig. 9. Growth rate for hyperbolic secant squared profile, w = 2000 km, H = 50 km, with friction.

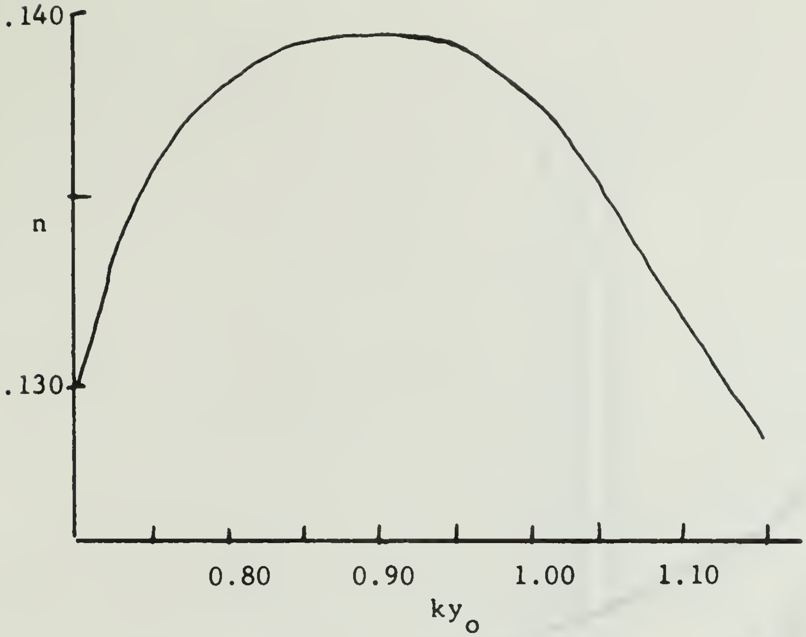

Fig. 10. Growth rate for hyperbolic secant squared profile, w = 2000 km, H = 25 km, with friction.

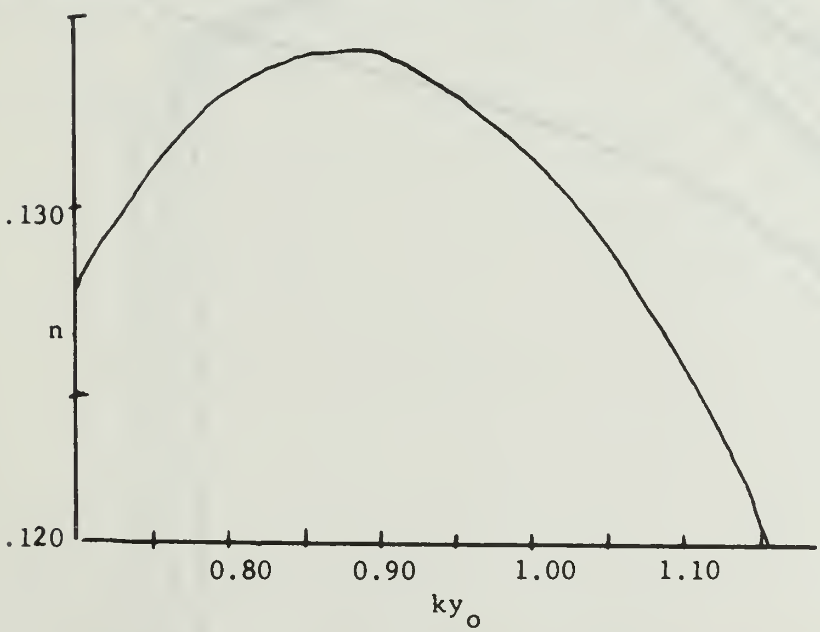

Fig. 11. Growth rate for hyperbolic secant squared profile, w = 4000 km, H = 25 km, with friction.

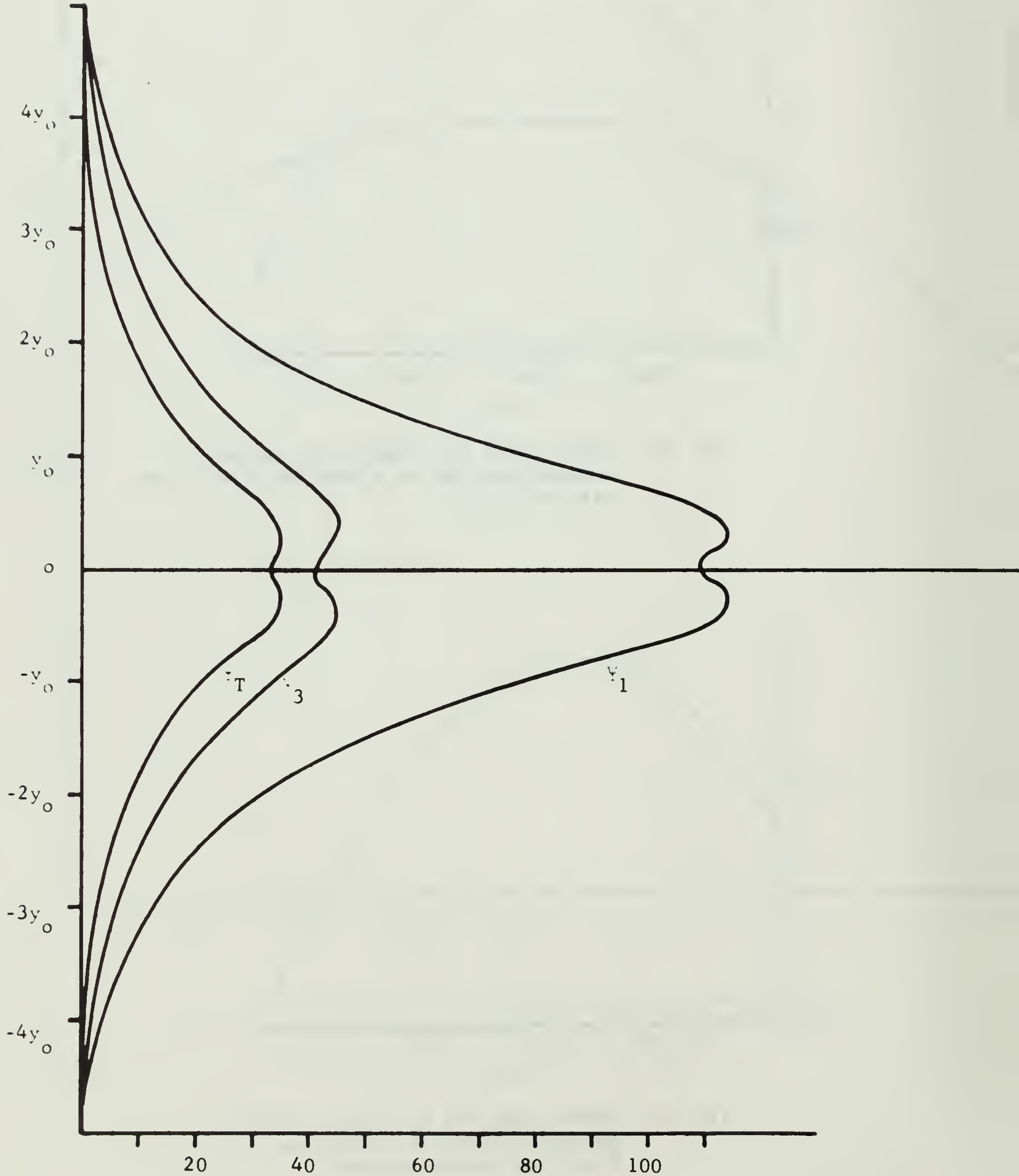

Fig. 12. Amplitudes of Ψ_T, Ψ_1, and Ψ_3 for hyperbolic tangent profile plotted against arbitrary scale, w = 4000 km, H = 50 km, and wave number = 0.45.

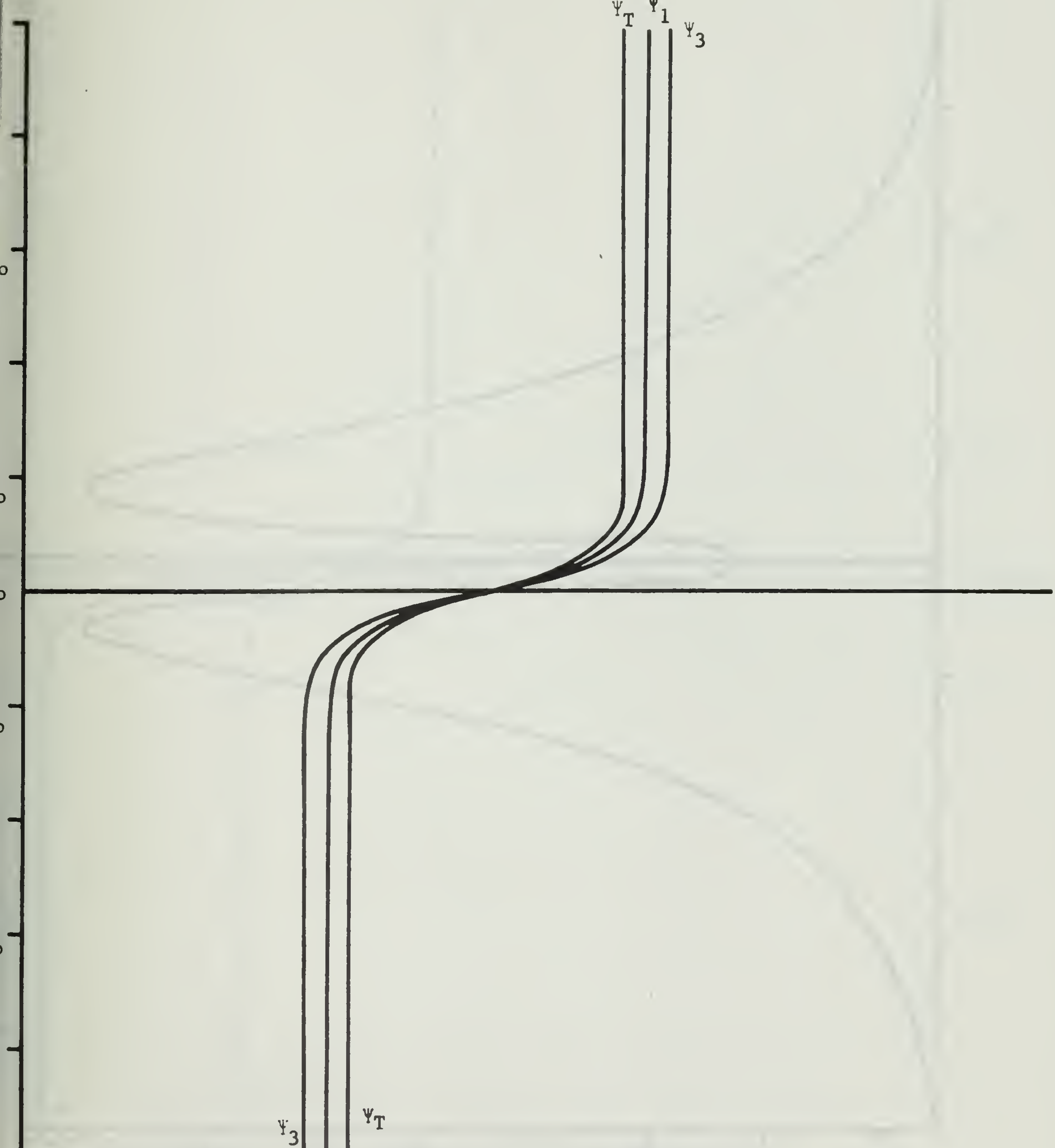

Fig. 13. Phase relationships corresponding to amplitudes of fig. 12.

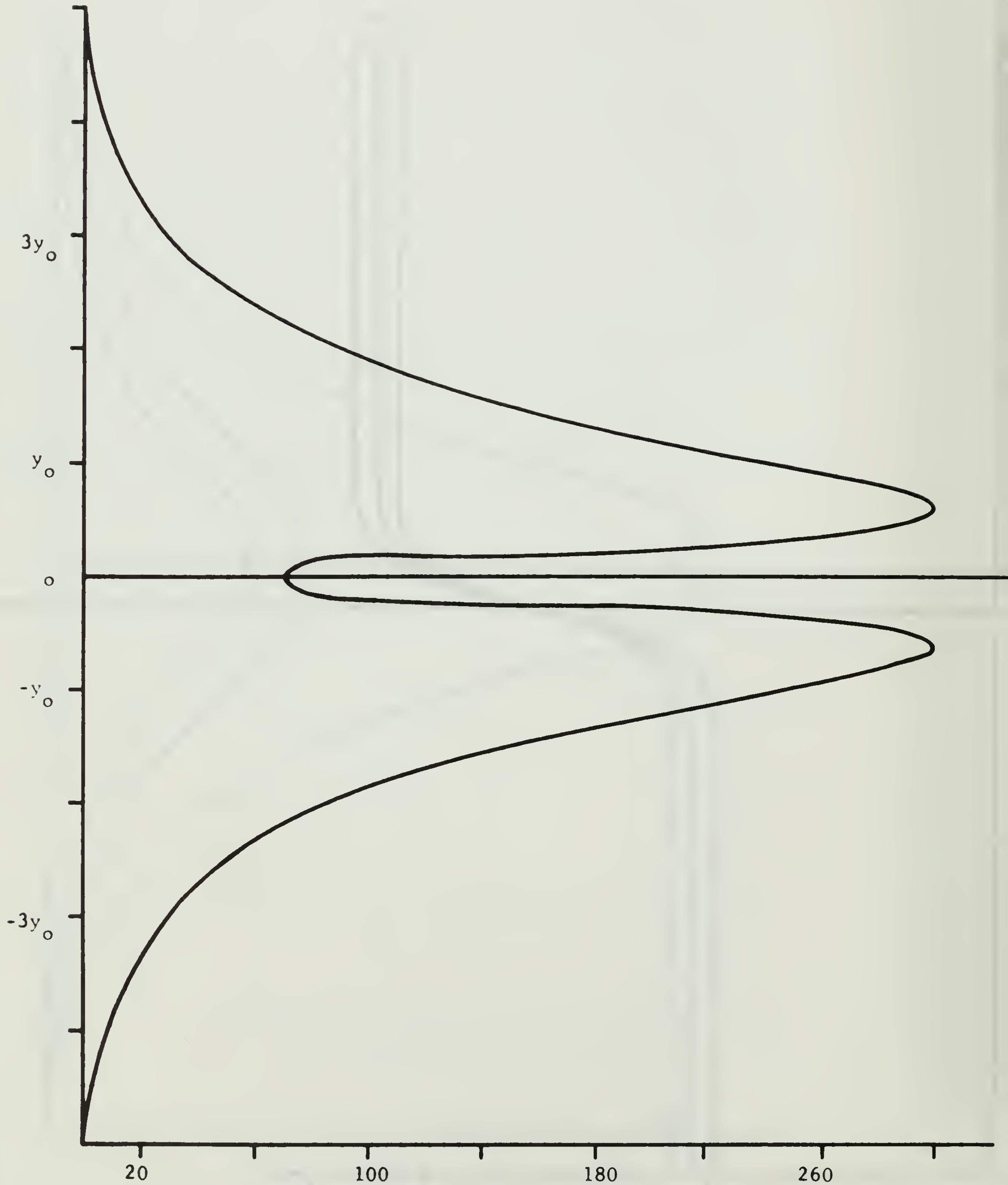

Fig. 14. Amplitude of omega, arbitrary scale for same case as figs. 12 and 13.

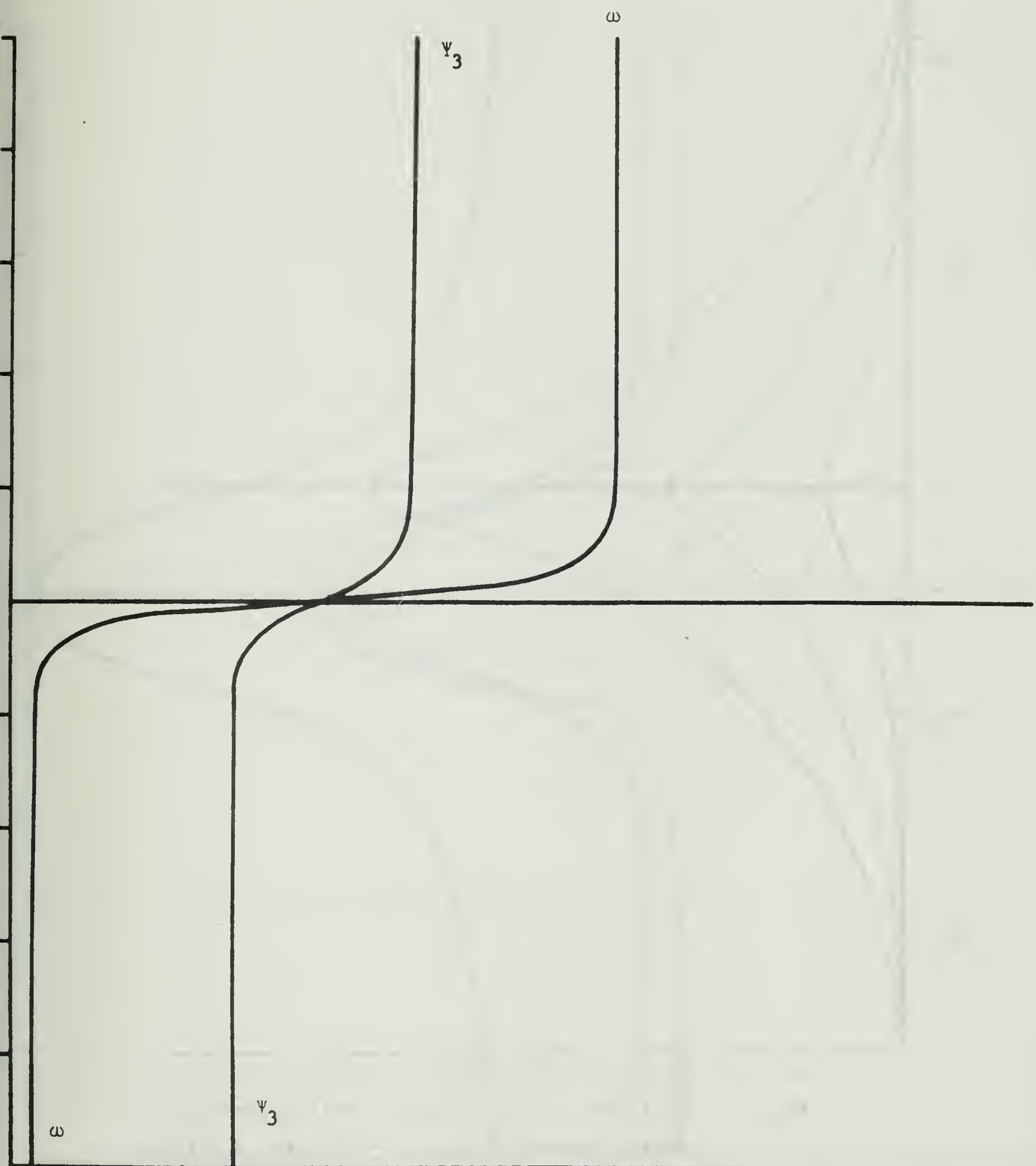

Fig. 15. Phase relationship between Ψ_3 and ω, same conditions as figs. 12, 13, and 14.

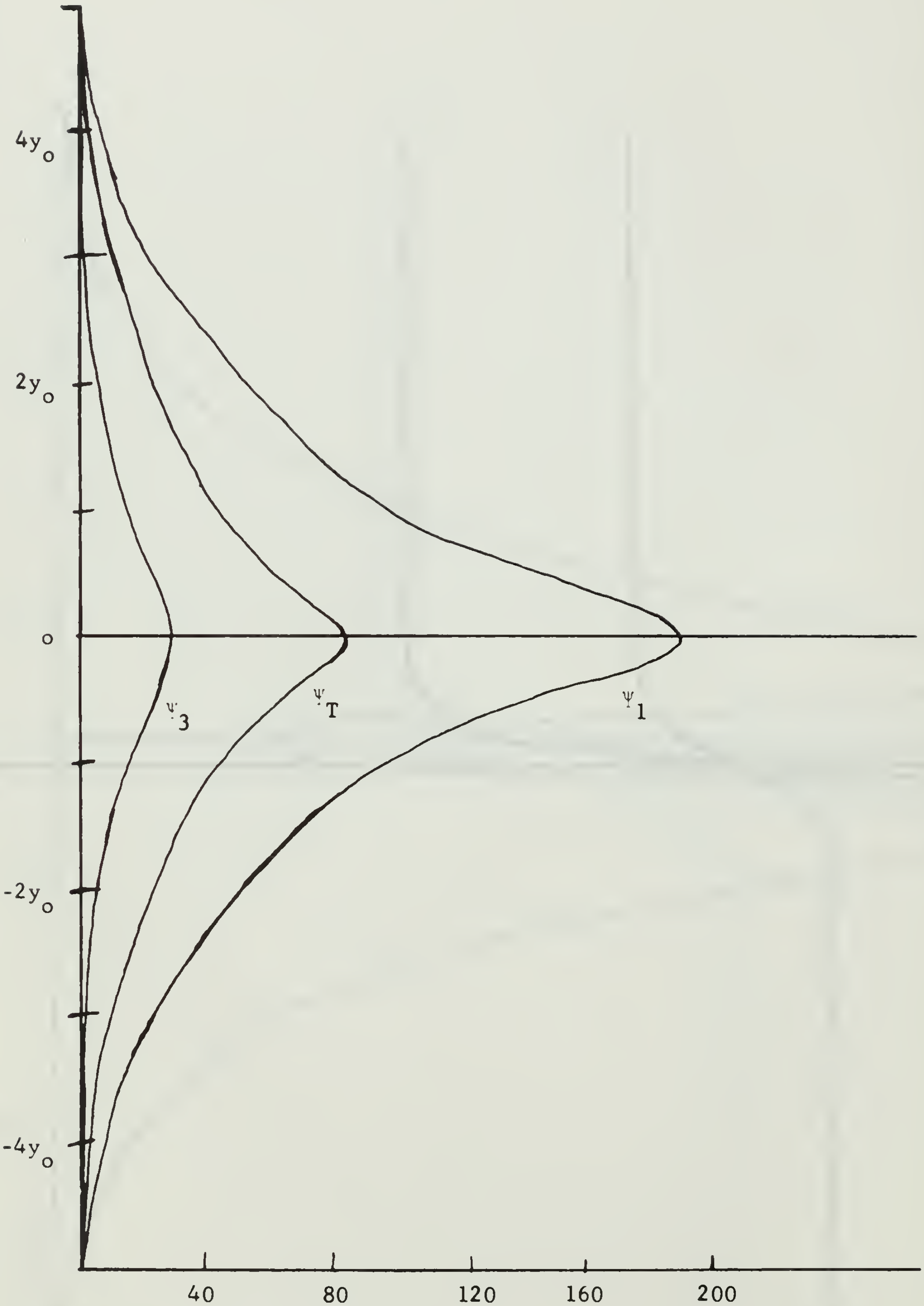

Fig. 16. Amplitude of Ψ_1, Ψ_3, and Ψ_T with hyperbolic secant squared profile, w = 2000 km, H = 25 km, ky_o = 0.95, and scale arbitrary.

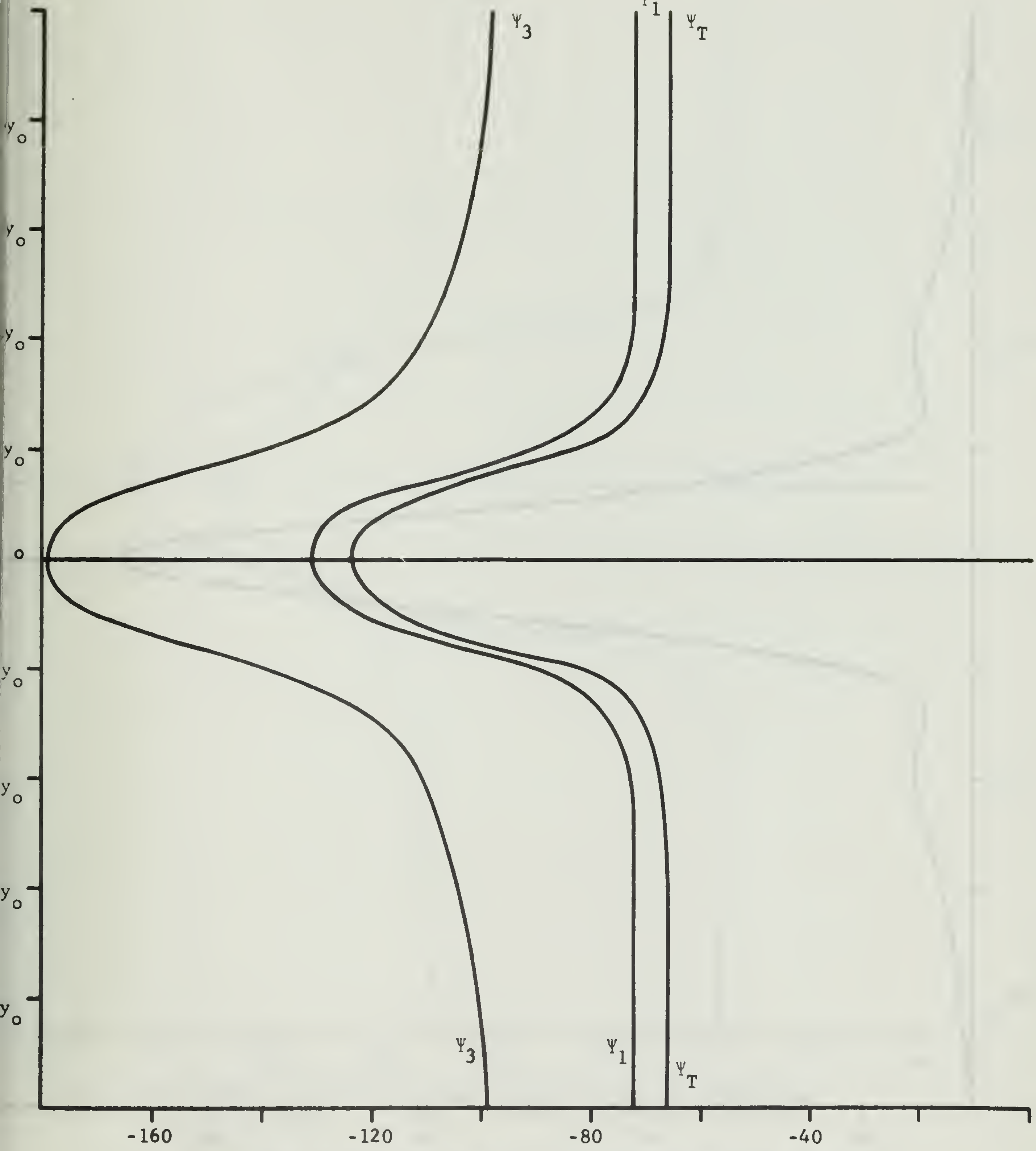

Fig. 17. Phase relationship for Ψ_1, Ψ_3, Ψ_T for same conditions as fig. 16.

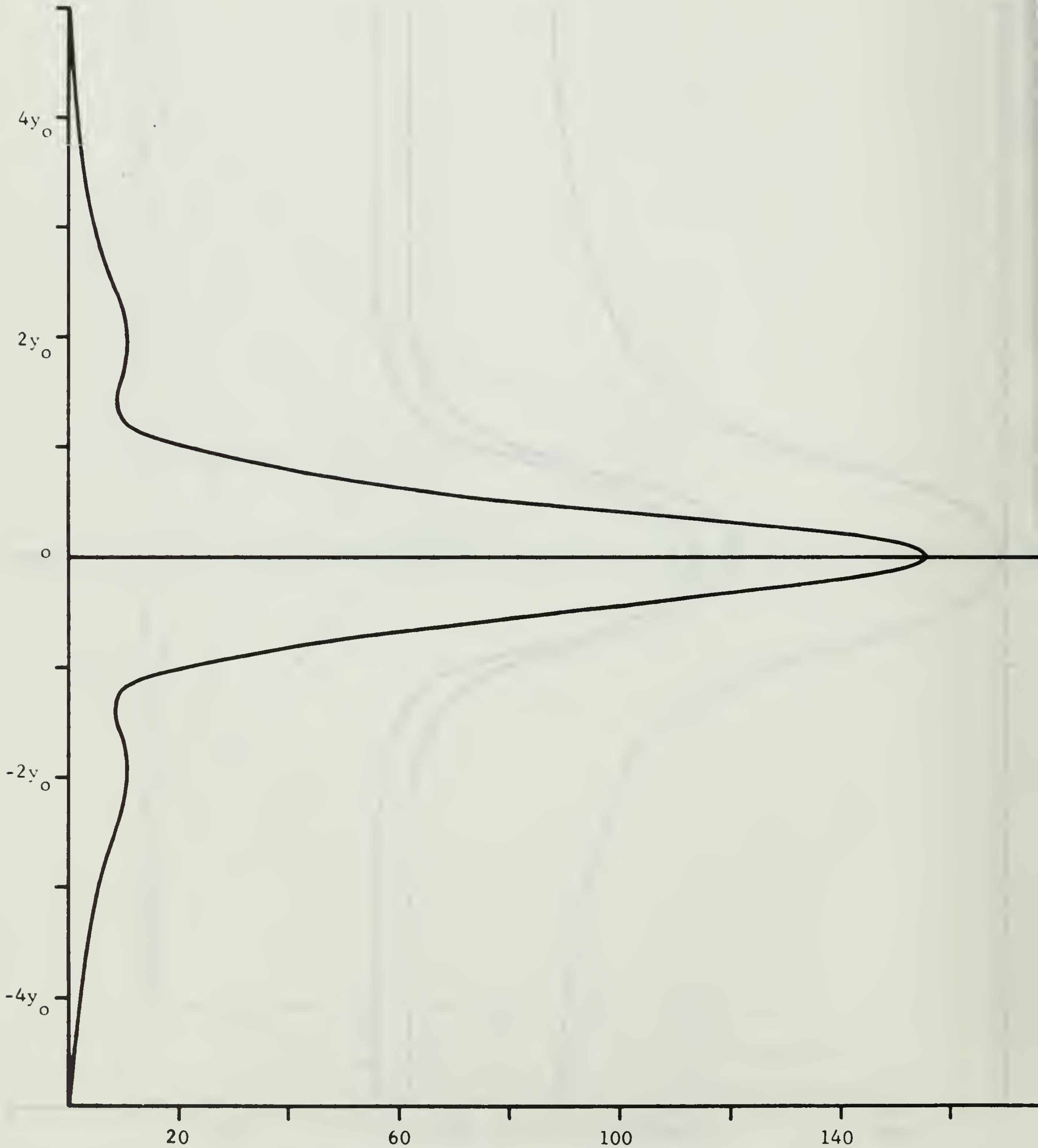

Fig. 18. Amplitude of omega, conditions as in fig. 16.

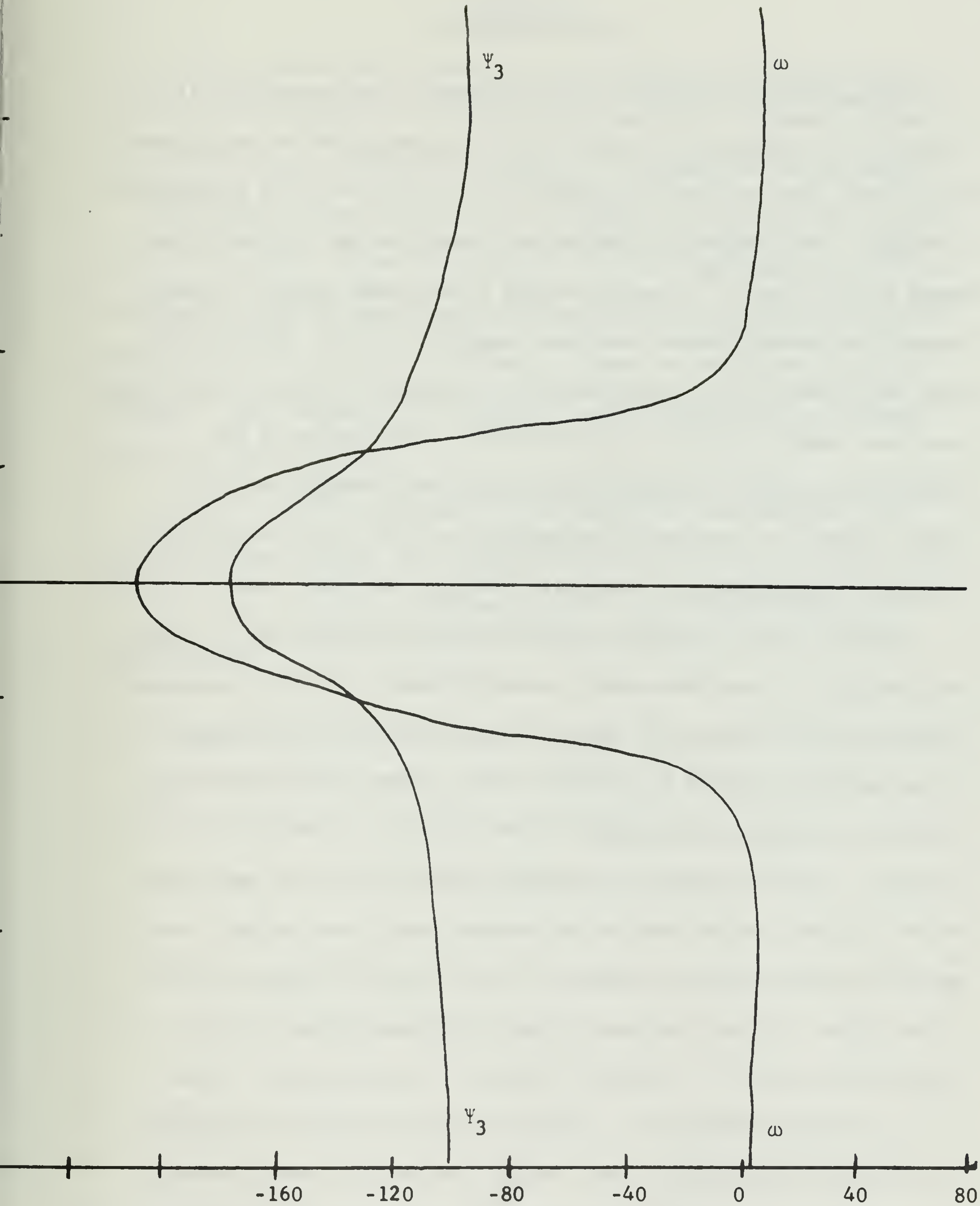

Fig. 19. Phase relationship between omega and Ψ_3 for conditions in fig. 16.

8. CONCLUSIONS

The easterly wave model developed appears to be reasonable if the assumption of barotropic instability as the energy source is accepted. Basic zonal flow has been introduced by two profiles, both independent of height. The temperature variations arose from the action of the Ekman friction terms. Waves developed by the model behaved in many respects as easterly waves have been observed to act. Results show that the growth rate of disturbances is a function of wave number, therefore wave length, as well as other factors. Separation of the boundaries to greater distances made some small changes but the greater effect is obtained by decreasing the grid size, which also required changing the time step size, through the shear zone.

Vertical motions produced by the hyperbolic tangent profile are very similar to those observed in easterly waves, in that the maximum sinking motion is upwind of the lower level ridge and the maximum rising motion is upwind of the lower level trough. The disturbance maintains its phase relationship in the vertical so that no tilt is observed. With the hyperbolic secant squared profile the amplitude of the vertical motion outside the maximum shear zone is small and again the rising motion is upwind of the trough and sinking upwind of the ridge. Within the area of large horizontal shear the convergent area moves to a position downwind of the low level trough. It is not known whether this is observationally verified along the Intertropical Convergence Zone. Using this profile, the hyperbolic secant squared, the model has developed a solution that indicates a definite eastward tilt with height. This tilt has been verified by observations. Perhaps some combination of the two profiles would be more realistic in depicting actual waves.

A more complete investigation of the effect of friction on the symmetric secant squared profile appears warranted. There is, also, a definite need to include the effects of condensation in the model.

Non-linear effects, their role in changing the mean wind profile and the structure of the disturbance, are very important and must be considered. Use of the primitive equations or the balance equations would seem to offer the most effective means of incorporating these effects. They appear to be crucial if an easterly wave is to grow into a tropical storm.

Always of utmost importance is the need for more detailed and extensive observations, but nowhere is this as true as of the tropics. No model can be developed and adequately tested without detailed observations.

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APPENDIX

```
C          PROGRAM IS A TWO LEVEL BAROTROPIC NUMERICAL WHICH USES A
C        CENTERED TIME INTEGRATION ON THE LINEARIZED EQUATIONS OF MOTION.
C        PROGRAM HAS A MAXIMUM OF 100 DATA POINTS INCLUDING BOUNDARIES
C        CONSTANTS AND VARIABLES READ IN FOR EACH RUN ARE
C        CORIOLIS, CHANGE IN CORIOLIS, SIGMA, BASIC PRESSURE AND GRID
C        DISTANCE
C        WAVE LENGTH IS COMPUTED BY THE PROGRAM
C
         DIMENSION ATDPA(100),BTDPA(100),CTDPA(100),DTDPA(100),ETDPA(100),
        *FTDPA(100),ATDPR(100),BTDPR(100),CTDPR(100),DTDPR(100),ETDPR(100),
        *FTDPR(100),ATDF(100),BTDF(100),CTDF(100),DTDF(100),ETDF(100),FTDF(
        *100),AEQ(100),BEQ(100),CEQ(100),DEQ(100),EEQ(100),FEQ(100),ASY(100
        *),BSY(100),CSY(100),DSY(100),ESY(100),FSY(100),ADY(100),BDY(100)
C
         DIMENSION CDY(100),DDY(100),EDY(100),FDY(100),ATY(100),BTY(100),CT
        *Y(100),DTY(100),ETY(100),FTY(100),GLOB(800),
        *WCOS (100),WSIN(100)
C
         COMMON    N,L,M,H,HI,TH,TCUH,HSQ,SQH,SQK,SQMU,FSQK,FSQMU,FUK,TFUK,
        2 BETAK,PKMU,SKMU,TIME,TIMAJ,ANNN,VB,SA,SB,SC,SD,SE,SF,NXT,RK,PIE
        3 ,CONS1,S
         COMMON ATDPA,      CTDPA,DTDPA,ETDPA,FTDPA,BTDPA,
        2       ATDPR,BTDPR,CTDPR,DTDPR,ETDPR,FTDPR,
        3       ATDF ,BTDF ,CTDF ,DTDF ,ETDF ,FTDF ,
        4AEQ,BEQ,CEQ,DEQ,EEQ,FEQ,ASY,BSY,CSY,DSY,ESY,FSY,
P       5 ADY,BDY,CDY,DDY,EDY,FDY,ATY,BTY,CTY,DTY,ETY,FTY,WCOS,WSIN
C
C        TO GET PRINT OUT OF ANY ERRORS ON IBM 360 COMPUTER
         CALL EP
C        TO GET EXECUTION TIME
         CALL SETIME
         NEWMAN = 0
C        INTRODUCE CORIOLIS AS ZERO FOR FIRST RUN, VARY IT LATER.
         PETA = 0.0
C
C        READ IN DATA
C        DATA READ IN IS USED TO CONTROL LENGTH OF TIME PROGRAM RUNS AND
C         PRINT OUT FREQUENCY.
         READ(5,4) NRUN,NPRT,N,FEPRT,TIME,AA
       4 FORMAT(3I6,3F7.2)
C        AA IS THE AMPLITUDE
       9 WRITE(6,12)
      12 FORMAT(1H0,5X,45HCONSTANTS AND VARIABLES USED IN THIS RUN ARE:,//)
         WRITE(6,6) NRUN,NPRT
       6 FORMAT(1H0,23H LENGTH OF RUN, NRUN = ,I4,44H INTEGRATIONS,  PRINT
        1OUT FREQUENCY, NPRT = ,I3,15H INTEGRATIONS. )
         FEOUT = FLOAT(NPRT) * FEPRT
         WRITE(6,8) FEOUT
```

```
    8 FORMAT(1H0,23H FE OUTPUT FREQUENCY = ,F5.0,13H INTEGRATIONS)
      WRITE(6,7)N,AA,TIME
    7 FORMAT(1H0,16H GRID SIZE, N = ,I3,27H POINTS,    AMPLITUDE, AA = ,F
     16.2,21H     TIME STEP SIZE = ,F10.2,9H SECONDS.)
C
C     CONSTANTS USED ARE
C
C     THIS SECTION DEFINES COMPUTED CONSTANTS USED THROUGHOUT PROGRAM
C     N IS THE NUMBER OF POINTS IN THE ONE DIMENSIONAL ARRAY.
C     ANNN IS THE NUMBER OF INTERVALS BETWEEN THE N POINTS.
      ANNN = N-1
      WRITE(6,602) ANNN
  602 FORMAT(4X,7HANNN = ,F7.2)
      FNOT = 0.000049879999
      SIGMA = 0.80
      H = 25000.0
      PNOT = 100.0
C     TOTAL DISTANCE FROM BOUNDARY TO BOUNDARY
      W = FLOAT(N-1)  * H
      WRITE(6,955) W
  955 FORMAT(1H0,42H TOTAL DISTANCE W BETWEEN THE N POINTS IS ,F12.2,8H
     1METERS.)
C     IN THE PROGRAM NOTATION DEL Y IS H
      YZERO = 8.0 * H
C     RK IS AREAL VALUE OF K BUT IS THE NON-DIMENSIONAL WAVE NUMBER
      RK = 11.0/(10.0 * YZERO)
      N1=((N-1) / 2) +1
      UNOT=10.0
      PIE=3.1415926535
      IDA=86400
      IHOR=3600
      ITIME = TIME
      ANL = (2.0*PIE)/RK
      WRITE(6,15) FNOT,BETA,SIGMA
   15 FORMAT(1H0,5X,7HFNOT = ,F15.12,5X,7HBETA = ,F18.15,5X,8HSIGMA = ,F
     115.12)
      WRITE(6,13) ANL,H,PNOT
   13 FORMAT(1H0,15HWAVELENGTH L = ,F10.0,40H METERS,    DISTANCE BETWEEN
     1 POINTS, H = ,F10.0,18H METERS,    PNOT = ,F6.2,11H CENTIBARS.)
      AKAY = 2.0*PIE / ANL
      SQK = AKAY*AKAY
      FNOTSQ = FNOT*FNOT
      SQPNOT=PNOT * PNOT
      SQMU = ( 8.0 * FNOTSQ ) / ( SQPNOT * SIGMA )
      FSQK = FNOT*SQK
      FSQMU = FNOT*SQMU
      FUK = AKAY / FNOT
      TFUK = AKAY / 2.0*FNOT
```

```
      PKMU = SQK + SQMU
      SKMU = SQK - SQMU
      BETAK = BETA*AKAY
      HI = 1.0 /H
      HSQ = H * H
      TH = 1.0 / (2.0*H)
      SQH = 1.0 /(H*H)
      TCUH = TH*SQH
      WCONST = 4.0 /(PNOT * SIGMA)
      ALPHA = PIE/W
C     FRICTION TERMS
      AAA = 10.0
      GRAVTY = 9.8
      RCON = 287.0
      ABSTMP = 300.0
      CONS1 = ((FNOT*GRAVTY)/(RCON*ABSTMP*2.0))* SQRT( AAA/FNOT )
      WRITE(6,16) CONS1
   16 FORMAT(1H0,10H CONS1 =  ,E16.8)
C     ALLOWANCE IS MADE FOR HEATING TERMS
      ATEA = 2.0
      DTETA = 0.0
C     HEATING ADDED  DENOTED AS S
      S = (SQMU*GRAVTY*DTETA*ATEA*SQRT( AAA/FNOT ) )/(8.0*ABSTMP*FNOT)
      S = PNOT * S
      WRITE(6,18) S
   18 FORMAT(1H0,5H S = ,E16.8)
C
C
C     INITIALIZE INDEXES
C
C     THE FOLLOWING ARE COUNTERS FOR:
C     END OF THE PROGRAM, WHERE NXT IS THE NUMBER OF INTEGRATIONS.
      NXT=0
C     PRINT OUT FREQUENCY
      NXTPRT = 1
C     PRINT OUT OF FE RELATIVE TO PRINT OUT FREQUENCY
      FECNT=1.0
C     TEST2 IS FOR THE FIRST TIME STEP DATA SHIFT.
      TEST2 = 1.0
C     TEST3 IS FOR RETURN FROM PRINTING ORIGINAL DATA.
      TEST3 = 1.0
C
C     BOUNDARY VALUES
C
C     THESE VALUES NEVER CHANGE.  OPERATE FROM L TO M NOT 1 TO N
C
C     THIS SECTION PUTS THE VALUE INTO ALL PRESENT TIME LOCATIONS
C     LESS THE BOUNDARIES.
```

```
C
      L=2
      M=N-1
C
C     TO TEST THE EQUATIONS SET THE FOLLOWING VALUES,
C
      A = 0.0
      B = 0.0
      C=0.0
      D=0.0
      E=0.0
      F=0.0
C
C     OPERATING VALUES
 1000 FORMAT(F15.12)
      DO 101 I = 1,N
      READ(5,1000) A
      ATDPR(I) = A
  101 CONTINUE
      DO 102 I=1,N
      READ(5,1000) B
      BTDPR(I) = B
  102 CONTINUE
      DO 103 I=1,N
      CTDPR(I) = C
  103 CONTINUE
      DO 104 I=1,N
      DTDPR(I) = D
  104 CONTINUE
      DO 105 I=1,N
      Y = H * FLOAT(I-N1)
      E =-(FNOT*UNOT*YZERO) * TANH(Y/YZERO)
      ETDPR(I) = E
  105 CONTINUE
      DO 106 I=1,N
      FTDPR(I) = F
  106 CONTINUE
C     FOR THE LOWER BOUNDARY VALUES
      I=1
      GO TO 24
C     FOR THE UPPER BOUNDARY
   25 I=N
   24 ATDPR(I) = 0.0
      BTDPR(I) = 0.0
      CTDPR(I) = 0.0
      DTDPR(I) = 0.0
      FTDPR(I) = 0.0
      ASY(I) = 0.0
```

```
      BSY(I) = 0.0
      CSY(I) = 0.0
      DSY(I) = 0.0
      ESY(I) = 0.0
      FSY(I) = 0.0
      ADY(I) = 0.0
      BDY(I) = 0.0
      CDY(I) =0.0
      DDY(I) =0.0
      EDY(I) = 0.0
      FDY(I) = 0.0
      IF ( I .EQ. 1 ) GO TO 25
C
C     PUT THE SAME DATA IN THE PAST AND PRESENT ARRAYS.
C
      DO 810 I=1,N
      ATDPA(I) = ATDPR(I)
      BTDPA(I) = BTDPR(I)
      CTDPA(I) = CTDPR(I)
      DTDPA(I) = DTDPR(I)
      ETDPA(I) = ETDPR(I)
      FTDPA(I) = FTDPR(I)
C
C     NOW FOR TEST SET FUTURE = PRESENT FOR E AND F
C
      ATDF(I) = ATDPR(I)
      BTDF(I) = BTDPR(I)
      CTDF(I) = CTDPR(I)
      DTDF(I) = DTDPR(I)
      ETDF(I) = ETDPR(I)
      FTDF(I) = FTDPR(I)
  810 CONTINUE
C
      DO 400 I = L,M
      ADY(I) = (ATDPR(I+1) + ATDPR(I-1) -2.0*ATDPR(I))  * SQH
      BDY(I) = (BTDPR(I+1) + BTDPR(I-1) - 2.0 * BTDPR(I)) * SQH
      CDY(I) = (CTDPR(I+1) + CTDPR(I-1) - 2.0 * CTDPR(I)) * SQH
      DDY(I) = (DTDPR(I+1) + DTDPR(I-1) - 2.0 * DTDPR(I)) * SQH
  400 CONTINUE
C
C     TO LABEL THE FIRST PAGE OF VARIABLES WITH TEST RELATIONSHIPS WE
      CALL ENERGY
C
C     TO PRINT OUT BASIC DATA
C
      GO TO 909
  950 TEST3 = 3.0
C
```

```
C     FIRST TIME INTEGRATION
C
C     THIS CUTS TIME STEP IN HALF FOR FIRST FORECAST TIME.
C
      TIMAJ = TIME / 2.0
      CALL FCST
      NCOUNT = 1
      GO TO 800
C
C     RESET TIME STEP FOR REMAINING OPERATIONS
C
  803 TIMAJ = TIME
      NXTPRT = 1
      TEST2 = 3.0
C
C     MAIN LOOP FOR FORECASTING
C
  900 CONTINUE
      CALL FCST
      NCOUNT = NCOUNT + 1
C
C     THIS INSURES THAT COMPUTE OMEGA ONLY AT TIMES DESIRED
C
      IF(NCOUNT.LT.336) GO TO 800
      DO 10 I =1,N
      WCOS(N-I+1) = WCONST *((CTDF(N-I+1)-CTDPA(N-I+1))/(2.0*TIME) + FUK
     2*((FSY(N-I+1) * BTDPR(N-I+1))-(ESY(N-I+1) * DTDPR(N-I+1))))
     3-WCONST*((S/SQMU)*((ADY(N-I+1)-CDY(N-I+1))-SQK*(ATDPA(N-I+1)-
     4CTDPR(N-I+1))))
      WSIN(N-I+1) = WCONST * ((DTDF(N-I+1)- DTDPA(N-I+1))/(2.0 * TIME) +
     2FUK*((ESY(N-I+1)* CTDPR(N-I+1)) -FSY(N-I+1) * ATDPR(N-I+1)))
     3-WCONST*((S/SQMU)*((BDY(N-I+1)-DDY(N-I+1))-SQK*(BTDPA(N-I+1)-
     4DTDPA(N-I+1))))
   10 CONTINUE
C
C     NOW SHIFT THE DATA, MOVING THE PRESENT DATA INTO THE
C     PAST ARRAY AND THE FUTURE DATA INTO THE PRESENT ARRAY.
C
  800 DO 801 I=1,N
      ATDPA(I) = ATDPR(I)
      BTDPA(I) = BTDPR(I)
      CTDPA(I) = CTDPR(I)
      DTDPA(I) = DTDPR(I)
      ETDPA(I) = ETDPR(I)
      FTDPA(I) = FTDPR(I)
  801 CONTINUE
      DO 802 I=1,N
      ATDPR(I) = ATDF(I)
```

```
      BTDPR(I) = BTDF(I)
      CTDPR(I) = CTDF(I)
      DTDPR(I) = DTDF(I)
      ETDPR(I) = ETDF(I)
      FTDPR(I) = FTDF(I)
  802 CONTINUE
C
C     INCREMENT COUNTERS
C
      NXT = NXT + 1
      NXTPRT = NXTPRT+ 1
C
C     TEST FOR PRINTOUT
C
  903 IF( NXTPRT .GE. NPRT) GO TO 909
      IF ( TEST2 .LT. 2.0 ) GO TO 803
C
C     TEST FOR END OF RUN
C
  902 IF(NXT .LT. NRUN) GO TO 900
C
C     WE HAVE REACHED THE MAX NUMBER OF INTEGRATIONS DESIRED, AND WE
C
C     GO TO 901
C
C     PRINT OUT OF DATA
C
C     COMPUTE THE HEADINGS FOR THE OUTPUT.
C
  909 IOPUT = NXT * ITIME
      IDAY = IOPUT / IDA
      IHRLFT = IOPUT - ( IDAY * IDA )
      IHOUR = IHRLFT / IHOR
      ISEC = IHRLFT - ( IHOUR * IHOR )
      WRITE(6,910) NXT,IDAY,IHOUR,ISEC
  910 FORMAT(1H1,    19H INTEGRATION NUMBER,I4,25H,  A TOTAL LAPSED TIME
     1 OF,I4,6H DAYS,I3,7H HOURS,,I4,9H SECONDS.,/)
      WRITE(6,922)
  922 FORMAT(126H  I                 A                    B                    C
     1                    D                    E                    F
     2     I,/)
  932 FORMAT(1H ,I3,3X,E16.8,3X,E16.8,3X,E16.8,3X,E16.8,3X,E16.8,3X,E16.
     28,3X,I3)
      DO 930 I = 1,N
      IK = N-I+1
      WRITE(6,932) IK,ATDPR(N-I+1),BTDPR(N-I+1),CTDPR(N-I+1),
     *DTDPR(N-I+1),ETDPR(N-I+1),FTDPR(N-I+1),IK
  930 CONTINUE
```

```
C
C     NOW CALCULATE THE ENERGY RELATIONSHIPS OF THE MODEL
C
      CALL ENERGY
      FECNT = FECNT + 1.0
C     FEPRT IS RELATIVE TO THE NUMBER OF TIMES PRINTOUT OCCURS
      IF(FECNT .LT. FEPRT ) GO TO 911
      WRITE(6,222)
  222 FORMAT(1HO,115H I   AMPLITUDE OF A AND B,    PHASE,A,B      AMPLITUDE
     2E OF C AND D,    PHASE2,C,D           WAMP                PHASE3,WC,WS )
C
C     COMPUTE PHASE ANGLE IN DEGREES
C
      WAMP = 0.0
      CAMP1 = 0.0
      CAMP2 = 0.0
      PHASE = 0.0
      PHASE2 = 0.0
      PHASE3 = 0.0
      PHASE4 = 0.0
      PHASE5 = 0.0
      FEAMP = 0.0
      FEAMP2 = 0.0
      NASSUM = 0
C
C     FAKE IS USED TO PREVENT DIVISION BY ZERO ON THE BOUNDARIES
C
      FAKE = 0.1E-19
      DO 220 I =1,N
      IJ = N-I+1
      IF( I .EQ. 1 ) GO TO 292
      IF( I .EQ. N ) GO TO 292
C     ESY(I) IS CALCULATED IN ENERGY AND DOES NOT HAVE TO BE REPEATED.
      WC            = WCONST *((CTDF(N-I+1)-CTDPA(N-I+1))/(2.0*TIME) + FUK
     2*((FSY(N-I+1) * BTDPR(N-I+1))-(ESY(N-I+1) * DTDPR(N-I+1))))
     3-WCONST*((S/SQMU)*((ADY(N-I+1)-CDY(N-I+1))-SQK*(ATDPA(N-I+1)-
     4CTDPR(N-I+1))))
      WS            = WCONST * ((DTDF(N-I+1)- DTDPA(N-I+1))/(2.0 * TIME) +
     2FUK*((ESY(N-I+1)* CTDPR(N-I+1)) -FSY(N-I+1) * ATDPR(N-I+1)))
     3-WCONST*((S/SQMU)*((BDY(N-I+1)-DDY(N-I+1))-SQK*(BTDPA(N-I+1)-
     4DTDPA(N-I+1))))
      IF(ABS(WC).GT. 0.0) GO TO 298
  299 PHASE3 = ATAN(WS/FAKE)
      NASSUM = NASSUM + 1
      GO TO 297
  298 PHASE3 = ATAN(WS/WC)
  297 IF((ABS(PHASE3)-0.8).GT.0.0) GO TO 170
      WAMP = WC / COS ( PHASE3 )
```

```
      GO TO 175
  170 WAMP = WS / SIN( PHASE3 )
  175 PHASE3 = 57.29578   * PHASE3
      IF(WAMP .GE. 0.0 ) GO TO 292
      IF( PHASE3 .GT. 0.0 ) GO TO 195
      PHASE3 = PHASE3 + 180.0
      GO TO 198
  195 PHASE3 = PHASE3 - 180.0
  198 WAMP =-WAMP
C
  292 IF(ABS(CTDPR(N-I+1)).GT.0.0) GO TO 120
  296 PHASE2 = ATAN(DTDPR(N-I+1)/FAKE)
      NASSUM = NASSUM + 1
      GO TO 295
  120 PHASE2 = ATAN(DTDPR(N-I+1)/CTDPR(N-I+1))
  295 IF ((ABS( PHASE2)- 0.8) .GT. 0.0 ) GO TO 140
      CAMP2 = CTDPR(N-I+1)/COS(PHASE2)
      GO TO 145
  140 CAMP2 = DTDPR(N-I+1)/SIN(PHASE2)
  145 PHASE2 = 57.29578   *PHASE2
      IF ( CAMP2 .GE. 0.0 ) GO TO 291
      IF ( PHASE2 .GT. 0.0 ) GO TO 165
      PHASE2 = PHASE2 + 180.0
      GO TO 168
  165 PHASE2 = PHASE2 - 180.0
  168 CAMP2 = -CAMP2
  291 IF(ABS(ATDPR(N-I+1)) .GT. 0.0) GO TO 117
  294 PHASE = ATAN(BTDPR(N-I+1)/FAKE)
      NASSUM = NASSUM + 1
      GO TO 293
  117 PHASE = ATAN(BTDPR(N-I+1)/ATDPR(N-I+1))
  293 IF((ABS(PHASE)-0.8).GT. 0.0) GO TO 90
   80 CAMP1 = ATDPR(N-I+1)/COS(PHASE)
      GO TO 95
   90 CAMP1 = BTDPR(N-I+1)/SIN(PHASE)
   95 PHASE = 57.29578    * PHASE
      IF( CAMP1 .GE. 0.0 ) GO TO 119
      IF ( PHASE .GT. 0.0 ) GO TO 115
  112 PHASE = PHASE + 180.0
      GO TO 118
  115 PHASE = PHASE - 180.0
  118 CAMP1 = - CAMP1
  119 WRITE(6,221)IJ,CAMP1,PHASE,CAMP2,PHASE2,WAMP,PHASE3,IJ
  221 FORMAT(3X ,I3,3X,E16.8,3X,E16.8,3X,E16.8,3X,E16.8,3X,E16.8,3X,E16.
     *8,3X,I3)
  220 CONTINUE
      WRITE(6,393) NASSUM
  393 FORMAT(3X,24HNUMBER OF ASSUMPTIONS = ,I4)
```

```
      FECNT = 0.0
  911 IF( TEST3 .LT. 2.0 ) GO TO 950
      IF ( TEST2 .LT. 2.0 ) GO TO 803
      NXTPRT = 0
C
C     TO TEST FOR THE END OF THE PROGRAM WE
C
      GO TO 902
  901 CONTINUE
C
C     PHASE4 IS FEE1 CASE,  A+C,  B+D
C     PHASE5 IS FEE3 CASE, A-C,  B-C
C
      WRITE(6,258)
  258 FORMAT(1H1,73H I   AMPLITUDE FEE1      PHASE FEE1       AMPLITUDE
     *FEE3          PHASE3   ,/)
      NASSUM = 0.0
      DO 257 I =1,N
      IJ = N-I+1
      FE8 = ATDPR(N-I+1) - CTDPR(N-I+1)
      FE9 = BTDPR(N-I+1) - DTDPR(N-I+1)
      IF(ABS(FE8) .GT. 0.0) GO TO 398
  399 PHASE5 = ATAN(FE9/FAKE)
      NASSUM = NASSUM + 1
      GO TO 397
  398 PHASE5 = ATAN( FE9 / FE8 )
  397 IF(( ABS( PHASE5 ) - 0.8 ) .GT. 0.0 ) GO TO 329
      FEAMP2= FE8 / COS( PHASE5 )
      GO TO 325
  329 FEAMP2= FE9 / SIN( PHASE5 )
  325 PHASE5 = 57.29578    * PHASE5
      IF( FEAMP2 .GE. 0.0 ) GO TO 355
      IF( PHASE5 .GT. 0.0 ) GO TO 345
      PHASE5 = PHASE5 + 180.0
      GO TO 348
  345 PHASE5 = PHASE5 - 180.0
  348 FEAMP2= -FEAMP2
  355 FE6 = ATDPR(N-I+1) + CTDPR(N-I+1)
      FE7 = BTDPR(N-I+1) + DTDPR(N-I+1)
      IF(ABS(FE6) .GT. 0.0) GO TO 395
  396 PHASE4 = ATAN(FE7/FAKE)
      NASSUM = NASSUM + 1
      GO TO 394
  395 PHASE4 = ATAN( FE7 / FE6 )
  394 IF(( ABS( PHASE4 ) - 0.8 ) .GT. 0.0 ) GO TO 229
      FEAMP = FE6 / COS( PHASE4 )
      GO TO 225
  229 FEAMP = FE7 / SIN( PHASE4 )
```

```
  225 PHASE4 = 57.29578   * PHASE4
      IF ( FEAMP .GE. 0.0 ) GO TO 255
      IF( PHASE4 .GT. 0.0 ) GO TO 245
      PHASE4 = PHASE4 + 180.0
      GO TO 248
  245 PHASE4 = PHASE4 - 180.0
  248 FEAMP = -FEAMP
  255 WRITE(6,256)IJ,FEAMP,PHASE4,FEAMP2,PHASE5
  256 FORMAT(3X ,I3,3X,E16.8,3X,E16.8,3X,E16.8,3X,E16.8)
  257 CONTINUE
      WRITE (6,393) NASSUM
C
C     PRINT OUT MAP FORM OF CALCULATIONS
C
C
C     FE ONE = FE M + FE T
C
      DO 300 I=1,N
      AEQ(I) = ATDPR(I) + CTDPR(I)
      BEQ(I) = BTDPR(I) + DTDPR(I)
  300 CONTINUE
      WRITE(6,310)
  310 FORMAT(1H1,24H FE ONE  = FE M  +  FE T)
      CALL MAP
C
C     FE THREE = FE M  -  FE T
C
      DO 301 I=1,N
      AEQ(I) = ATDPR(I) - CTDPR(I)
      BEQ(I) = BTDPR(I) - DTDPR(I)
  301 CONTINUE
      WRITE(6,311)
  311 FORMAT(1H1,24H FE THREE= FE M  -  FE T)
      CALL MAP
C
C     MEAN FLOW ADDED TO MOTION FOR FE THREE
C
      DO 320 I=1,N
      ETDPR(I) = ETDPR(I) + FNOT*H*5.0*FLOAT(I)
  320 CONTINUE
      WRITE(6,321)
  321 FORMAT(1H1,33H FE THREE MODIFIED BY FNOT*H*5*I )
      CALL MAP
      DO 322 I=1,N
      ETDPR(I) = 0.0
  322 CONTINUE
C
C     OMEGA FIELD = WC + WS
```

```
C
      DO 302 I=1,N
      AEQ (N-I+1) = WCONST *((CTDF(N-I+1)-CTDPA(N-I+1))/(2.0*TIME) + FUK
     2*((FSY(N-I+1) * BTDPR(N-I+1))-(ESY(N-I+1) * DTDPR(N-I+1))))
     3-WCONST*((S/SQMU)*((ADY(N-I+1)-CDY(N-I+1))-SQK*(ATDPA(N-I+1)-
     4CTDPR(N-I+1))))
      BEQ (N-I+1) = WCONST * ((DTDF(N-I+1)- DTDPA(N-I+1))/(2.0 * TIME) +
     2FUK*((ESY(N-I+1)* CTDPR(N-I+1)) -FSY(N-I+1) * ATDPR(N-I+1)))
     3-WCONST*((S/SQMU)*((BDY(N-I+1)-DDY(N-I+1))-SQK*(BTDPA(N-I+1)-
     4DTDPA(N-I+1))))
  302 CONTINUE
      WRITE(6,312)
  312 FORMAT(1H1,20H OMEGA  =  WC  +  WS)
      CALL MAP
C
C     FEM AND FET  REVISITED
C     FE M WITH A,B
C
      DO 303 I=1,N
      AEQ(I) = ATDPR(I)
      BEQ(I) = BTDPR(I)
  303 CONTINUE
      WRITE(6,313)
  313 FORMAT(1H1,20H FE M WITH A,B      )
      CALL MAP
      DO 304 I=1,N
      AEQ(I) = CTDPR(I)
      BEQ(I) = DTDPR(I)
  304 CONTINUE
      WRITE(6,314)
  314 FORMAT(1H1,20H FE T WITH C,D      )
      CALL MAP
      NEWMAN = NEWMAN + 1
      IF(NEWMAN.LT.2) GO TO 2
      GO TO 3
    2 BETA = 2.29E-11
      WRITE(6,5)
    5 FORMAT(2X,36H*  *   BETA IS REAL FOR ALL THE REST)
      GO TO 9
    3 CALL GETIME (IET)
      XX = IET * 0.000026
      WRITE(6,1) XX
    1 FORMAT(3X,17HTIME FOR RUN IS  ,E16.6,7HSECONDS)
      STOP
      END
```

```
      SUBROUTINE ENERGY
C
C     THIS ROUTINE KEEPS TRACK OF THE ENERGY OF THE MODEL
C
      DIMENSION ATDPA(100),BTDPA(100),CTDPA(100),DTDPA(100),ETDPA(100),
     *FTDPA(100),ATDPR(100),BTDPR(100),CTDPR(100),DTDPR(100),ETDPR(100),
     *FTDPR(100),ATDF(100),BTDF(100),CTDF(100),DTDF(100),ETDF(100),
     *FTDF(100),AEQ(100),BEQ(100),CEQ(100),DEQ(100),EEQ(100),FEQ(100),
     *ASY(100),BSY(100),CSY(100),DSY(100),ESY(100),FSY(100),ADY(100)
      DIMENSION BDY(100),CDY(100),DDY(100),EDY(100),FDY(100),ATY(100),
     *BTY(100),CTY(100),DTY(100),ETY(100),FTY(100),GLOB(800)
      COMMON    N,L,M,H,HI,TH,TCUH,HSQ,SQH,SQK,SQMU,FSQK,FSQMU,FUK,TFUK,
     2 BETAK,PKMU,SKMU,TIME,TIMAJ,ANNN,VB,SA,SB,SC,SD,SE,SF,NXT,RK,PIE
     3 ,CONS1,S
      COMMON ATDPA,      CTDPA,DTDPA,ETDPA,FTDPA,BTDPA,
     2       ATDPR,BTDPR,CTDPR,DTDPR,ETDPR,FTDPR,
     3       ATDF ,BTDF ,CTDF ,DTDF ,ETDF ,FTDF ,
     4AEQ,BEQ,CEQ,DEQ,EEQ,FEQ,ASY,BSY,CSY,DSY,ESY,FSY,
     5 ADY,BDY,CDY,DDY,EDY,FDY,ATY,BTY,CTY,DTY,ETY,FTY
      WRITE(6,21)
   21 FORMAT(1H0,111H A = RANDOM     , B = RANDOM     , C = 0      , D = 0
     *        , E = TANH(Y/YO), F = 0 , MEAN MAPS,FCT, N41,STEST57)
C
C     FOR THE SUMS OF ENERGY WE INITIALIZE
C
      SASY = 0.0
      SBSY = 0.0
      SCSY = 0.0
      SDSY = 0.0
      SESY = 0.0
      SFSY = 0.0
      SA = 0.0
      SB = 0.0
      SC = 0.0
      SD = 0.0
      SE = 0.0
      SF = 0.0
C
C      AND THE L TO M IS VALID,FOR THE FIRST AND LAST POINTS ARE 0
C
      DO 800 I=L,M
C
C     SINGLE DERIVATIVES FOR A POINT
C
      ASY(I)=(ATDPR(I+1)-ATDPR(I-1))*TH
      BSY(I)=(BTDPR(I+1)-BTDPR(I-1))*TH
      CSY(I)=(CTDPR(I+1)-CTDPR(I-1))*TH
      DSY(I)=(DTDPR(I+1)-DTDPR(I-1))*TH
```

```
      ESY(I)=(ETDPR(I+1)-ETDPR(I-1))*TH
      FSY(I)=(FTDPR(I+1)-FTDPR(I-1))*TH
C
C     SUM OF THE ENERGY CALCULATIONS
C
      SASY = SASY + ASY(I) * ASY(I)
      SBSY = SBSY + BSY(I) * BSY(I)
      SCSY = SCSY + CSY(I) * CSY(I)
      SDSY = SDSY + DSY(I) * DSY(I)
      SESY = SESY + ESY(I) * ESY(I)
      SFSY = SFSY + FSY(I) * FSY(I)
C
C     ADJUST THE SUMS FOR THE ENERGY EQUATIONS
C
      SA = SA + ATDPR(I) * ATDPR(I)
      SB = SB + BTDPR(I) * BTDPR(I)
      SC = SC + CTDPR(I) * CTDPR(I)
      SD = SD + DTDPR(I) * DTDPR(I)
      SE = SE + ETDPR(I) * ETDPR(I)
      SF = SF + FTDPR(I) * FTDPR(I)
  800 CONTINUE
C
C     USING A ONE SIDED DIFFERENCE SCHEME ALONG THE BOUNDARIES
C
      I = L
      GO TO 810
  809 I = N
  810 ASY(I) = ( ASY(I) - ASY(I-1) )  *  HI
      BSY(I) = ( BSY(I) - BSY(I-1) )  *  HI
      CSY(I) = ( CSY(I) - CSY(I-1) )  *  HI
      DSY(I) = ( DSY(I) - DSY(I-1) )  *  HI
      ESY(I) = ( ESY(I) - ESY(I-1) )  *  HI
      FSY(I) = ( FSY(I) - FSY(I-1) )  *  HI
C
C     COMPLETEION OF THE SUMS
C
      SASY = SASY + (.5 * ASY(I) * ASY(I)  )
      SBSY = SBSY + (.5 * BSY(I) * BSY(I)  )
      SCSY = SCSY + (.5 * CSY(I) * CSY(I)  )
      SDSY = SDSY + (.5 * DSY(I) * DSY(I)  )
      SESY = SESY + (.5 * ESY(I) * ESY(I)  )
      SFSY = SFSY + (.5 * FSY(I) * FSY(I)  )
      IF ( I .EQ. L ) GO TO 809
C
C     COLLECT ENERGY
C        DISTURBANCE KINETIC ENERGY   OR DKE
C
      DKE=(0.25 *(SQK*(SA + SB + SC + SD ) + ( SASY + SBSY + SCSY +
```

```
     1 SDSY ) )   ) / ANNN
C
C     DISTURBANCE POTENTIAL ENERGY OR DPE
C
      DPE =(SQMU * 0.25 * (SC + SD) ) / ANNN
C
C     MEAN FLOW KINETIC ENERGY OR ENMKE
C
      ENMKE = ( 0.5 * ( SESY + SFSY )  )  / ANNN
C
C     MEAN FLOW POTENTIAL ENERGY OR ENMPE
C
      ENMPE = ( SQMU * 0.5 * SF ) / ANNN
C
C     TOTAL ENERGY IS
C
      ENED = DKE + DPE
      ENEM = ENMKE + ENMPE
      TOTE = ENED + ENEM
      WRITE(6,24) DKE,DPE,ENMKE,ENMPE
   24 FORMAT(1H0,6H DKE =,E16.8,7H, DPE =,E16.8,9H, ENMKE =,E16.8,9H, EN
     2MPE =,E16.8)
      WRITE(6,20) ENED,ENEM,TOTE
   20 FORMAT(1H0,22H DISTRUBANCE ENERGY =,E16.8,20H MEAN FLOW ENERGY = ,
     *E16.8,19H TOTAL ENERGY IS = ,E16.8)
      RETURN
      END



      SUBROUTINE RCHMYR(XEQ,XTDPR,XTDPA,XTDF)
C
C     CALCULATES EXACT SOLUTION FOR THE DATA FIELD
C
      DIMENSION ATDPA(100),BTDPA(100),CTDPA(100),DTDPA(100),ETDPA(100),
     *FTDPA(100),ATDPR(100),BTDPR(100),CTDPR(100),DTDPR(100),ETDPR(100),
     *FTDPR(100),ATDF(100),BTDF(100),CTDF(100),DTDF(100),ETDF(100),
     *FTDF(100),AEQ(100),BEQ(100),CEQ(100),DEQ(100),EEQ(100),FEQ(100),
     *ASY(100),BSY(100),CSY(100),DSY(100),ESY(100),FSY(100),ADY(100)
      DIMENSION BDY(100),CDY(100),DDY(100),EDY(100),FDY(100),ATY(100),
     *BTY(100),CTY(100),DTY(100),ETY(100),FTY(100),VD(100),VE(100),
     *VF(100),DVX(100),XEQ(100),XTDPA(100),XTDPR(100),XTDF(100)
      COMMON   N,L,M,H,HI,TH,TCUH,HSQ,SQH,SQK,SQMU,FSQK,FSQMU,FUK,TFUK,
     2 BETAK,PKMU,SKMU,TIME,TIMAJ,ANNN,VB,SA,SB,SC,SD,SE,SF,NXT,RK,PIE
     3 ,CONSI,S
      COMMON ATDPA,     CTDPA,DTDPA,ETDPA,FTDPA,BTDPA,
     2         ATDPR,BTDPR,CTDPR,DTDPR,ETDPR,FTDPR,
```

```
     3        ATDF  ,BTDF  ,CTDF  ,DTDF  ,ETDF  ,FTDF  ,
     4AEQ,BEQ,CEQ,DEQ,EEQ,FEQ,ASY,BSY,CSY,DSY,ESY,FSY,
     5 ADY,BDY,CDY,DDY,EDY,FDY,ATY,BTY,CTY,DTY,ETY,FTY
C
C         BOUNDARY CONDITIONS
C
      VD(1) = 0.0
      VD(N) = 0.0
      VE(1) = 0.0
      VE(N) = 0.0
      VF(1) = 0.0
      VF(N) = 0.0
      DVX(1) = 0.0
      DVX(N) = 0.0
C
C              RICHTMYER SOLUTION
C     CALCULATE COEFFICIENTS FOR NEXT STEP
C
      DO 450 I=L,M
      VD(I) = -2.0*HSQ*TIMAJ* XEQ(I)
      VE(I) = 1.0 / ( VB - VE(I-1) )
      VF(I) = ( VD(I) + VF(I-1))* VE(I)
  450 CONTINUE
C
C     CALCULATES VALUES FOR EACH POINT.
C
      DO 454 I = 1,M
      DVX(M-I+1) = (VE(M-I+1) * DVX(M-I+2)) + VF(M-I+1)
  454 CONTINUE
C
C  RCHMYR TECHNIQUES ENDS HERE.  NEXT WE PUT VALUES INTO THE FUTURE ARRAY
C
      DO 458 I=1,N
      XTDF(I) = XTDPA(I) + DVX(I)
  458 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MAP
      DIMENSION ATDPA(100),BTDPA(100),CTDPA(100),DTDPA(100),ETDPA(100),
     *FTDPA(100),ATDPR(100),BTDPR(100),CTDPR(100),DTDPR(100),ETDPR(100),
     *FTDPR(100),ATDF(100),BTDF(100),CTDF(100),DTDF(100),ETDF(100),
     *FTDF(100),AEQ(100),BEQ(100),CEQ(100),DEQ(100),EEQ(100),FEQ(100),
     *ASY(100),BSY(100),CSY(100),DSY(100),ESY(100),FSY(100),ADY(100)
      DIMENSION BDY(100),CDY(100),DDY(100),EDY(100),FDY(100),ATY(100),
     *BTY(100),CTY(100),DTY(100),ETY(100),FTY(100),GLOB(800)
      COMMON    N,L,M,H,HI,TH,TCUH,HSQ,SQH,SQK,SQMU,FSQK,FSQMU,FUK,TFUK,
     2 BETAK,PKMU,SKMU,TIME,TIMAJ,ANNN,VB,SA,SB,SC,SD,SE,SF,NXT,RK,PIE
     3 ,CONS1,S
      COMMON ATDPA,      CTDPA,DTDPA,ETDPA,FTDPA,BTDPA,
     2       ATDPR,BTDPR,CTDPR,DTDPR,ETDPR,FTDPR,
     3       ATDF ,BTDF ,CTDF ,DTDF ,ETDF ,FTDF ,
     4AEQ,BEQ,CEQ,DEQ,EEQ,FEQ,ASY,BSY,CSY,DSY,ESY,FSY,
     5 ADY,BDY,CDY,DDY,EDY,FDY,ATY,BTY,CTY,DTY,ETY,FTY
      DO 301 I= 1,N
      DO 302 J=1,11
      RKX = ( PIE/10.0) * FLOAT ( J-1 )
      ASY(J) = (AEQ(N-I+1) * COS(RKX) +BEQ(N-I+1)* SIN(RKX)) * 0.2 + ETD
     *PR(N-I+1)
  302 CONTINUE
      IJ = N-I+1
      WRITE(6,304)IJ,(ASY(J),J=1,11),IJ
  304 FORMAT(1H ,I3,11(E11.3),3X,I3,/)
  301 CONTINUE
      WRITE(6,320)
  320 FORMAT(1H1)
      DO 311 I = 1,N
      DO 312 J=11,21
      RKX = ( PIE/10.0) * FLOAT ( J-1 )
      ASY(J) = (AEQ(N-I+1) * COS(RKX) +BEQ(N-I+1)* SIN(RKX)) * 0.2 + ETD
     *PR(N-I+1)
  312 CONTINUE
      IJ = N-I+1
      WRITE(6,314)IJ,(ASY(J),J=11,21),IJ
  314 FORMAT(1H ,I3,11(E11.3),3X,I3,/)
  311 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FCST
C
C     CALCULATES NEW VALUES BY FINITE DIFFERENCE AND CALLS RCHMYR.
C
      DIMENSION ATDPA(100),BTDPA(100),CTDPA(100),DTDPA(100),ETDPA(100),
     *FTDPA(100),ATDPR(100),BTDPR(100),CTDPR(100),DTDPR(100),ETDPR(100),
     *FTDPR(100),ATDF(100),BTDF(100),CTDF(100),DTDF(100),ETDF(100),
     *FTDF(100),AEQ(100),BEQ(100),CEQ(100),DEQ(100),EEQ(100),FEQ(100),
     *ASY(100),BSY(100),CSY(100),DSY(100),ESY(100),FSY(100),ADY(100)
      DIMENSION BDY(100),CDY(100),DDY(100),EDY(100),FDY(100),ATY(100),
     *BTY(100),CTY(100),DTY(100),ETY(100),FTY(100),GLOB(800)
      COMMON   N,L,M,H,HI,TH,TCUH,HSQ,SQH,SQK,SQMU,FSQK,FSQMU,FUK,TFUK,
     2 BETAK,PKMU,SKMU,TIME,TIMAJ,ANNN,VB,SA,SB,SC,SD,SE,SF,NXT,RK,PIE
     3 ,CONS1,S
      COMMON ATDPA,     CTDPA,DTDPA,ETDPA,FTDPA,BTDPA,
     2       ATDPR,BTDPR,CTDPR,DTDPR,ETDPR,FTDPR,
     3       ATDF ,BTDF ,CTDF ,DTDF ,ETDF ,FTDF ,
     4AEQ,BEQ,CEQ,DEQ,EEQ,FEQ,ASY,BSY,CSY,DSY,ESY,FSY,
     5 ADY,BDY,CDY,DDY,EDY,FDY,ATY,BTY,CTY,DTY,ETY,FTY
      GLOB(1) = 0.0
      GLOB(N) = 0.0
      GLOB(N+1) = 0.0
      GLOB(N+N) = 0.0
C
C     FRICTION TERMS
C
      DO 120 I=L,M
      GLOB(I) = ( ADY(I)-CDY(I) ) -SQK * ( ATDPA(I) - CTDPA(I) )
      GLOB( I+N) = (BDY(I)-DDY(I) ) - SQK*(BTDPA(I)-DTDPA(I) )
  120 CONTINUE
      DO 500 I=L,M
C
C     SINGLE DERIVATIVES FOR A POINT
C
      ASY(I)=(ATDPR(I+1)-ATDPR(I-1))*TH
      BSY(I)=(BTDPR(I+1)-BTDPR(I-1))*TH
      CSY(I)=(CTDPR(I+1)-CTDPR(I-1))*TH
      DSY(I) = (DTDPR(I+1)-DTDPR(I-1))* TH
      ESY(I)=(ETDPR(I+1)-ETDPR(I-1))*TH
      FSY(I)=(FTDPR(I+1)-FTDPR(I-1))*TH
C
C     DOUBLE DERIVATIVES FOR A POINT
C
      ADY(I) = (ATDPR(I+1) + ATDPR(I-1) - 2.0 * ATDPR(I)) * SQH
      BDY(I) = (BTDPR(I+1) + BTDPR(I-1) -2.0 * BTDPR(I)) * SQH
      CDY(I)=(CTDPR(I+1)+CTDPR(I-1)-2.0*CTDPR(I))*SQH
      DDY(I)=(DTDPR(I+1)+DTDPR(I-1)-2.0*DTDPR(I))*SQH
      EDY(I)=(ETDPR(I+1)+ETDPR(I-1)-2.0*ETDPR(I))*SQH
```

```
      FDY(I)=(FTDPR(I+1)+FTDPR(I-1)-2.0*FTDPR(I))*SQH
  500 CONTINUE
C
C     TO GET OVER BOUNDARY PROBLEMS OF TRIPLE DERIVATIVES, SET
C
      ADY(1) = 0.0
      BDY(1) = 0.0
      CDY(1) = 0.0
      DDY(1) = 0.0
      EDY(1) = 0.0
      FDY(1) = 0.0
      AEQ(1) = 0.0
      BEQ(1) =0.0
      CEQ(1) =0.0
      DEQ(1) = 0.0
      EEQ(1) = 0.0
      FEQ(1) = 0.0
      I=N
      ADY(I) = 0.0
      BDY(I) = 0.0
      CDY(I) = 0.0
      DDY(I) = 0.0
      EDY(I) = 0.0
      FDY(I) = 0.0
      AEQ(N) =0.0
      BEQ(N) =0.0
      CEQ(N) =0.0
      DEQ(N) =0.0
      EEQ(N) = 0.0
      FEQ(N) =0.0
C
C     TRIPLE DERIVATIVES FOR A POINT
C
      DO 510 I=L,M
      ATY(I) = ( ADY(I+1) - ADY(I-1) ) * TH
      BTY(I) = ( BDY(I+1) - BDY(I-1) ) * TH
      CTY(I) = ( CDY(I+1) - CDY(I-1) ) * TH
      DTY(I) = ( DDY(I+1) - DDY(I-1) ) * TH
      ETY(I) = ( EDY(I+1) - EDY(I-1) ) * TH
      FTY(I) = ( FDY(I+1) - FDY(I-1) ) * TH
C
C     DERIVATIVE EXPRESSIONS COLLECTED FOR THE TIME DERIVATIVE
C     FRICTION TERMS ADDED
C
      AEQ(I) = FUK*((SOK*((-ESY(I)*BTDPR(I))-(FSY(I)*DTDPR(I))))
     1 + (ESY(I)*BDY(I)) - (ETY(I)*BTDPR(I))+(FSY(I)*DDY(I))
     2 - (FTY(I)*DTDPR(I))) - BETAK*BTDPR(I)
     3 - CONS1 * GLOB(I)
```

```
      BEQ(I) = FUK*((SQK*(( ESY(I)*ATDPR(I))+(FSY(I)*CTDPR(I))))
     1 - (ESY(I)*ADY(I)) + (ETY(I)*ATDPR(I)) - (FSY(I)*CDY(I)) +
     2  (FTY(I)*CTDPR(I))) + BETAK*ATDPR(I)
     9 - CONS1 * GLOB( I+N )
      CEQ(I) = FUK*((SQK*(-FSY(I)*BTDPR(I))) - (ESY(I)*DTDPR(I)*PKMU)
     1+ (FSY(I)*BTDPR(I)*SQMU)+(ESY(I)*DDY(I))-(FTY(I)*BTDPR(I))
     2 +(FSY(I)*BDY(I)) - (ETY(I)*DTDPR(I))) - BETAK*DTDPR(I)
     9 + ( CONS1 - S ) * GLOB( I )
      DEQ(I) = FUK*((SQK*FSY(I)*ATDPR(I)) + (ESY(I)*PKMU*CTDPR(I))
     1 - (ESY(I)*CDY(I)) + (FTY(I)*ATDPR(I)) - (SQMU*FSY(I)*ATDPR(I))
     2 - (FSY(I)*ADY(I)) + (ETY(I)*CTDPR(I))) + BETAK*CTDPR(I)
     9 + ( CONS1 - S ) * GLOB( I+N )
      FEQ(I) = TFUK*( (ATDPR(I)*BDY(I)) - ( BTDPR(I)*ADY(I))
     1 + (CTDPR(I)*DDY(I)) - (DTDPR(I)*CDY(I)) )
      FEQ(I) = TFUK*( (ATDPR(I)*(DDY(I)-DTDPR(I)*PKMU)) - (BTDPR(I)*
     1 (CDY(I)-CTDPR(I)*PKMU)) + (CTDPR(I)*(BDY(I) -BTDPR(I)*SQK))
     2 -(DTDPR(I)*(ADY(I) - ATDPR(I)*SQK)) )
  510 CONTINUE
C
C     TIME DERIVATIVE STEP FOR A,B,C,D,E,F
C
C     CALL THE RICHTMYER SOLUTION
C
C     FOR AEQ AND BEQ, VB DOES NOT CHANGE
C
      VB = 2.0 + HSQ * SQK
      CALL RCHMYR (AEQ,ATDPR,ATDPA,ATDF)
      CALL RCHMYR(BEQ,BTDPR,BTDPA,BTDF)
C
C     FOR CEQ AND DEQ, VB DOES NOT CHANGE
C
      VB = 2.0 + HSQ*PKMU
      CALL RCHMYR (CEQ,CTDPR,CTDPA,CTDF)
      CALL RCHMYR(DEQ,DTDPR,DTDPA,DTDF)
C
C     FOR EEQ, VB IS
C
      VB = 2.0
C
C     FOR FEQ, VB IS
C
      VB = 2.0 + HSQ*SQMU
      RETURN
      END
```

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BSTRACT

A simple two-level numerical model using the quasi-geostrophic forecast equations is developed. Equations are linearized and friction is introduced in the surface layer. Solutions are obtained numerically by using the initial value approach. Two wind profiles, $U = -U_o \tanh y/y_o$ and $U = U_o \operatorname{sech}^2 y/y_o$, are used and these are known to be unstable. For each wind profile the growth rate is determined as a function of the wave number. Some observed features of easterly waves are reproduced in the numerical solutions.

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